DEPTH OF INVESTIGATION IN DIRECT CURRENT METHODS†

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The depth of investigation in any direct current resistivity method of surface geophysical prospecting is defined, following Evjen (1938), as that depth at which a thin horizontal (parallel to the ground surface) layer of ground contributes the maximum amount to the total measured signal at the ground surface. Using the equivalence between static and stationary fields, we have found the following values for the absolute depths of investigation in homogeneous ground:

<table>
<thead>
<tr>
<th>Electrode System</th>
<th>Depth of Investigation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two electrode</td>
<td>0.35L</td>
</tr>
<tr>
<td>Equatorial or azimuthal dipole ($\theta_1 = \pi/4$)</td>
<td>0.25L</td>
</tr>
<tr>
<td>Perpendicular dipole ($\theta_1 = \pi/4$)</td>
<td>0.20L</td>
</tr>
<tr>
<td>Polar or radial dipole ($\theta_1 = \pi/4$)</td>
<td>0.195L</td>
</tr>
<tr>
<td>Parallel dipole ($\theta_1 = \pi/4$)</td>
<td>0.18L</td>
</tr>
<tr>
<td>Modified unipole</td>
<td>0.18L</td>
</tr>
<tr>
<td>Surface laterolog ($x_0_2 = 0.1L$)</td>
<td>0.17L</td>
</tr>
<tr>
<td>Surface laterolog ($x_0_2 = 0.2L$)</td>
<td>0.135L</td>
</tr>
<tr>
<td>Schlumberger</td>
<td>0.125L</td>
</tr>
<tr>
<td>Wenner</td>
<td>0.11L</td>
</tr>
</tbody>
</table>

where L is the distance in any system between the two extreme active electrodes (that is, we disregard those electrodes at infinity, in case they exist). The most important points to note are that (i) the simplest nonfocused two-electrode system has by far the largest depth of investigation, (ii) focusing the current depthwards does not necessarily make a system superior with respect to its depth of investigation, (iii) the depth of investigation in any system is a good deal smaller than what is generally assumed, and (iv) the depth of investigation in any electrode system is determined by the positions of both the current and the potential electrodes and not by the current penetration or distribution alone.

The advantage of the two-electrode system in having a high depth of investigation is counterbalanced somewhat by its low vertical resolution. The various electrode systems can be arranged as follows in order of decreasing vertical resolution:

Wenner (highest vertical resolution)

Schlumberger

Parallel dipole, $\theta_1 = \pi/4$

Polar (or radial with $\theta_1 = \pi/4$) dipole

Perpendicular dipole, $\theta_1 = \pi/4$

Surface laterolog, $l = 0.2L$

Surface laterolog, $l = 0.1L$ and modified unipole

Equatorial (or azimuthal with $\theta_1 = \pi/4$) dipole

Two electrode (lowest vertical resolution).

The method of investigation developed in this paper is applicable to inhomogeneous ground also, as illustrated by its application to a two-layer model. For conducting vein ores, the superiority of the two-electrode system over the others is illustrated by some model tank curves.

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INTRODUCTION

Depth of investigation is an important physical concept in any method of geophysical prospecting. For fields describing signals that propagate, the depth of investigation has a quite precise and clear meaning; in a general way, different depths give rise to separate signals on the record. For methods using fields in which no signals propagate, the meaning becomes hazy in the sense that we can no longer pinpoint the depth of investigation; earth materials from all depths and distances contribute to the measured signal in varying degrees. Fortunately, in many of the artificial-field methods of this latter class, the contributions from the various earth layers at increasing depths do not fall off monotonically with increase in depth but pass through a maximum. Thus, we can still define the depth of investigation unambiguously as that depth which contributes most to the total signal measured on the ground surface. Some aspects of the problem of defining depth of investigation for electromagnetic induction methods have been examined by Doll (1949), Paul and Roy (1970), and Roy and Dhar (1970). In this paper, we treat direct current resistivity methods.

For a long time in direct current resistivity prospecting, the depth of investigation has been considered synonymous with the depth of current penetration or inferred from the current distribution in general, it being tacitly assumed that a higher percentage of current flowing deeper would necessarily mean a higher depth of investigation. If, for instance, a current of unit strength flows between two point electrodes, separated by a distance \( L \), on the horizontal surface of a homogeneous and isotropic half-space, the current density \( J_z \) on the vertical line through the midpoint between the electrodes is

\[
J_z |_{x=0} = \frac{1}{2\pi} \frac{1}{(L^2/4 + z^2)^{3/2}}, \tag{1}
\]

where the \( x \) axis is along the line joining the positive to the negative electrode, the \( z \) axis points downward, and the origin is at the midpoint. The fraction of the total current \( \Delta I/I \) flowing within a depth \( z \) from the ground surface across the plane \( x=0 \) is given by

\[
\Delta I/I = (2/\pi) \cdot \arctan (2z/L). \tag{2}
\]

Figure 1 graphs relations (1) and (2) for \( L = 2 \).
stances, not even approximately correct. Other variations of such assumptions exist; one instance is the assumption that the depths of interfaces in a layered model can be determined from the inflection points on the apparent resistivity curve by multiplication of the electrode separation by a numerical constant, a constant sometimes taken equal to unity.

The notions mentioned in the preceding paragraph persisted even though Evjen (1938), in a paper ahead of its time, gave clear reasons and made categorical statements against them. He was the first to have clearly defined the depth factor or depth of investigation more or less in the same way as given in the first paragraph of this paper. In an extremely interesting, although at times abstract, treatment based on image densities, Evjen found that for the usual Wenner electrode arrangement the depth of investigation is one-ninth, not one-third, of the distance between the current electrodes. Evjen’s result is identical to what we have obtained in this paper through a much simpler and physically evident approach.

Al’pin et al. (1966) considered the problem of depth of investigation for the specific case of a layered earth, the lowest layer being perfectly insulating. From a comparison of the separations at which the apparent resistivity sounding curves approach their asymptotes, Al’pin concluded that, for a given depth of investigation, the following relations hold:

\[ OQ = \frac{1}{3} AB \] for a radial dipole array,

\[ = \frac{1}{2} AB \] for a azimuthal dipole array,

\[ = \frac{3 \cos^2 \theta - 1}{2 \cos 2\theta} \] for a parallel dipole array,

\[ = \frac{3}{2} AB \] for a perpendicular dipole array,

where \( OQ \) is the distance between the transmitting and receiving dipoles, \( AB \) is the distance between the two current electrodes in a Schlumberger array, and \( \theta \) is the azimuthal angle for the dipole arrays. Relatively speaking, therefore, a radial dipole has the same depth of investigation as a Schlumberger arrangement; an azimuthal dipole has a depth of investigation twice as large as that of Schlumberger; and so on. From Figure 66, pages 118–119, in Keller and Frischknecht (1966), it would appear that the Wenner arrangement has a depth of investigation that is slightly smaller than that of the Schlumberger array. If the depth of investigation for the Schlumberger array is unity, that for a Wenner array is approximately 0.92.

Keller (1966) extended Al’pin’s investigations to cover the cases when (1) the dipole lengths in a polar dipole are not small compared to the separation and the lower layer is not completely insulating and (2) the dipole orientations are arbitrary. Keller also considered a three-layer case of low-high-low sequence and apparently came to the conclusion that the actual electrode separations for the same depth of investigation in case of the equatorial dipole, polar dipole, and the Schlumberger arrays need to be in the proportion of 1:1.2:2.

Fröhlich (1967) also considered three-layered sequences of low-high-low or high-low-high type and apparently came to the conclusion that the depth of investigation increases from Schlumberger to polar to perpendicular to equatorial dipole arrays. For the parallel array, the depth of investigation varies and can become superior or inferior to that from any of the above systems depending on the angle \( \theta \). In giving the above order of depths of investigation, we have, unlike Fröhlich, considered the entire separation \( AB \), not \( AB/2 \), for the Schlumberger arrangement, so that its depth of investigation is exactly half that of the equatorial dipole array, in keeping with relations (3).

**ELECTROSTATIC EQUIVALENCE**

Given an electrode system, we desire basically to find the contribution made by each individual

\[ \frac{\partial \rho^D}{\partial r} = \left( 1 - \frac{1}{e^2} \right) \frac{\partial \rho^S}{\partial r} - \frac{1}{e} \frac{\partial^2 \rho^S}{\partial r^2} \]

At the maximum of the Schlumberger arrangement, \( \partial \rho^S/\partial r \) is zero and \( \partial^2 \rho^S/\partial r^2 \) is negative. This makes \( \partial \rho^D/\partial r \) (slope of dipole curve) positive if \( e \) is positive.

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1 The denominator appearing in Al’pin et al. (1966) is 2 \( \cos^2 \theta \). This seems to be a misprint. In any case, the conclusions of our paper are not dependent on this difference.

2 "Apparently," because we are not sure we have understood Keller (1966) fully.

3 Fröhlich’s conclusions rest on a demonstration that the slope of the dipole curve is positive at the maximum given by the Schlumberger arrangement. A shorter demonstration seems possible. From Fröhlich’s equation (1), we can write

\[ \frac{\partial \rho^D}{\partial r} = \left( 1 - \frac{1}{e^2} \right) \frac{\partial \rho^S}{\partial r} - \frac{1}{e} \frac{\partial^2 \rho^S}{\partial r^2} \]

At the maximum of the Schlumberger arrangement, \( \partial \rho^S/\partial r \) is zero and \( \partial^2 \rho^S/\partial r^2 \) is negative. This makes \( \partial \rho^D/\partial r \) (slope of dipole curve) positive if \( e \) is positive.
elementary volume of ground to the total signal—a potential difference—measured on the ground surface. In order to do so, we imagine that the point electrodes emitting or receiving current $I$ are replaced by point charges of electricity of strength plus or minus $(pI/2\pi)$ placed on the free surface of a dielectric half-space. Provided we agree to measure electrostatic potentials instead of signals, such a mental exercise is permissible, since the stationary current regime in the actual ground and the electrostatic regime in its dielectric equivalent have identical properties. With such a transformation, each rectangular volume element of the dielectric will acquire three components of electrostatic polarization and will give rise to dipolar fields of its own. The dipolar electrostatic potential produced at the measuring point or points by the polarized volume element is the latter’s contribution to the total signal. When the contributions from all the volume elements are added, we should get the potential equivalent to the signal that we would measure or theoretically compute at the ground surface if we worked in the direct current regime.

Since we are concerned with depths of investigation, we will integrate the electrostatic potentials due to the induced dipoles only over planes parallel to the horizontal ground surface. These integrated values are the contributions to the total signal, measured at the potential electrodes on the ground surface, by the individual thin horizontal layers that together constitute the homogeneous half-space. The integrated values or individual contributions, when plotted against the corresponding depths of the thin horizontal layers, result in a curve that we will call the depth investigation characteristic for the particular system of electrodes being considered. The depth at which this curve attains a maximum is, according to our definition, the depth of investigation for that electrode system. Let it be noted that the individual contributions of the horizontal layers cannot actually be measured; what we observe on the ground surface between the potential measuring points is the total contribution from all the constituent layers given by the area under the depth investigation characteristic curve. Figure 2 shows most of the electrode systems analyzed below in this manner and their interrelationship through a common $L$.
DERIVATION OF FORMULAS

Homogeneous ground: Wenner, Schlumberger, and two-electrode arrays

With the xy plane coincident with the ground surface, the z axis pointing downwards, one current electrode of strength +I at (0, 0, 0), one potential electrode P₁ at (a, 0, 0), another potential electrode P₂ at (a+b, 0, 0), and the other current electrode of strength -I at (a+b+c, 0, 0), the potential at any point (x, y, z) in a homogeneous isotropic half-space of resistivity ρ is

\[
dV_{P₁P₂} = \frac{\rho I}{4\pi^2} \cdot \frac{dV}{dz} \left[ \frac{x(x-a) + y^2 + z^2}{(x^2 + y^2 + z^2)^{3/2}} \right. \\
- \frac{(x-a)(x-a-b-c) + y^2 + z^2}{(x-a-b+c)^2 + y^2 + z^2} \left. \right] dx dy dz.
\]

Integrating equation (7) over the xy plane from minus to plus infinity for both x and y (see Appendix for integration), we find the depth investigation characteristic (DIC) to be

\[
DIC = \int_{x=-\infty}^{x=\infty} \int_{y=-\infty}^{y=\infty} dV_{P₁P₂} = \frac{\rho I}{4\pi} \cdot \\
\left[ \frac{8\pi z}{(a^2 + 4z^2)^{3/2}} \right. \\
- \frac{8\pi z}{(b + c)^2 + 4z^2} \left. \right] dz.
\]

The dipole moments of the volume element at (x, y, z) will be proportional to the electric field at that point and can therefore be assumed to be

\[
\begin{align*}
\mu_x &= \frac{1}{2\pi} \cdot \frac{\partial V}{\partial x} \cdot dx \cdot dy dz, \\
\mu_y &= \frac{1}{2\pi} \cdot \frac{\partial V}{\partial y} \cdot dy \cdot dx dz, \\
\mu_z &= \frac{1}{2\pi} \cdot \frac{\partial V}{\partial z} \cdot dz \cdot dx dy,
\end{align*}
\]

in the x, y, and z directions. The justification for taking the proportionality factor as (1/2π) will soon follow.

The potential difference between P₁ and P₂ caused by the above dipole moments is

\[
dV_{P₁P₂} = \left[ \frac{\mu_x}{\partial x} + \frac{\mu_y}{\partial y} + \frac{\mu_z}{\partial z} \right] \cdot \left( \frac{1}{K₁} - \frac{1}{K₂} \right),
\]

where

\[
R₁ = [(x-a)^2 + y^2 + z^2]^{1/2},
\]

and

\[
R₂ = [(x-a-b)^2 + y^2 + z^2]^{1/2}.
\]

That is,
\[
V_{P_1P_2} = \int_{z=-\infty}^{z=\infty} \text{DIC} = \frac{\rho I}{2\pi} \left[ \frac{1}{a} - \frac{1}{b+c} - \frac{1}{a+b+c} \right], \quad (9)
\]
as it should be. The result obtained in equation (9) provides the justification for the assumptions made in equation (5), including the one for a constant of proportionality of \(1/2\pi\).

For the Wenner electrode arrangement, \(a = b = c = L/3\), where \(L\) is the distance between the two outermost active electrodes. Dividing by \(3\rho I/2\pi L\) (= total response of half-space), we compute the normalized depth investigation characteristic for this system from equation (8) to be

\[
\text{DIC}(N)_{\text{WENNER}} = dz \cdot \frac{8Lz}{3} \cdot \left( \frac{1}{(L^2/9 + 4z^2)^{3/2}} - \frac{1}{(4L^2/9 + 4z^2)^{3/2}} \right). \quad (10)
\]

It should be noted that, provided every \(2z\) is replaced by \(z\), relation (10) is identical with the function \(P_1(a, z)\) of Evjen (1938) derived through a completely different approach. For the Schlumberger array, let us take \(a = c = 0.45L\) and \(b = 0.1L\). The normalizing factor from equation (9) is \(3\rho I/2.475\pi L\). Therefore, from equation (8), we have

\[
\text{DIC}(N)_{\text{SCHL}} = dz \cdot 9.9Lz \cdot \left[ \frac{1}{(0.45L)^2 + 4z^2}^{3/2} - \frac{1}{(0.55L)^2 + 4z^2}^{3/2} \right]. \quad (11)
\]

For the two-electrode (normal or potential in well-logging parlance) system, \(b = c = \infty\) and \(a = L\), since \(a\) now is the distance between the two extreme active electrodes. The electrodes at infinity are disregarded in the definition of \(L\), since they do not contribute anything to the measurements. The normalizing factor is \(\rho I/2\pi a\) or \(\rho I/2\pi L\). Therefore, from equation (8) again,

\[
\text{DIC}(N)_{\text{T.E.}} = dz \cdot \frac{4Lz}{(L^2 + 4z^2)^{3/2}}. \quad (12)
\]

**Homogeneous ground: modified unipole array**

When both the current electrodes in the Schlumberger system are made positive and the sink is removed to infinity, we get what Gupta and Bhattacharya (1963) called the unipole, a focused system. By removing one potential electrode \(P_2\) also to infinity, one measures the potential at the midpoint of the array instead of the potential gradient. This modified unipole system, as we have called it, has the following advantages: (1) a profile across a conducting target now consists of one clear trough as against a peak-and-a-trough combination of the unipole, and (2) the anomaly magnitude is larger. In actual field use, the signal to be measured in the modified arrangement is also much larger.

For the modified unipole IPI array with a distance \(L\) between the two positive current electrodes and with the potential probe \(P\) located halfway between them, the third and the fourth terms within the squared brackets in expression (8) disappear; the second term changes sign; \(a = (b+c) = L/2\); and the normalizing factor becomes \((2\rho I/\pi L)\). Therefore,

\[
\text{DIC}(N)_{\text{MOD.UNIPOLE}} = dz \cdot \frac{2Lz}{(L^2/4 + 4z^2)^{3/2}}. \quad (13)
\]

**Homogeneous ground: surface laterolog**

The focused laterolog system was introduced in well logging by Doll (1951). Its surface equivalent (see also Apparao and Roy, 1969 and 1971) is shown in Figure 3. Current is fed into the ground by three electrodes of the same polarity: \(+I(0, 0, 0)\), \(+I(L/2, 0, 0)\), and \(+I(L, 0, 0)\). The relative magnitudes of \(I\) and \(I_0\) are adjusted until a null is obtained between the shorted pairs of potential electrodes \(P_1P'_1\) and \(P_2P'_2\). A focused sheet of current of inline thickness \(0_{10}\) flows into the ground and the potential at any one of the four electrodes \(P_1, P'_1, P_2,\) and \(P'_2\) under the null condition is measured as signal.

In order that a minimum of potential may occur at \(0_1\) and \(0_2\) (Figure 3), the following relation must hold:

\[
I_0 = \frac{4L^3}{(L^2 - L^2)^2} \cdot I. \quad (14)
\]
The potential $V_L$ at any point $(x, y, z)$ of the ground due to the three current electrodes is

$$V_L(x, y, z) = \frac{\rho I}{2\pi} \left[ \frac{1}{(x^2 + y^2 + z^2)^{1/2}} + \frac{4L^3}{(L^2 - l^2)^2} \right]$$

$$+ \frac{1}{(x - L/2)^2 + y^2 + z^2}^{1/2}$$

$$+ \frac{1}{(x - L)^2 + y^2 + z^2}^{1/2}.$$  \hspace{1cm} (15)

With expressions similar to those in equation (5) for components of dipole moments and with

$$R = [(x - (L \mp l)/2)^2 + y^2 + z^2]^{1/2}$$

as the distance between $O_1$ or $O_2$ and the volume element at $(x, y, z)$, the potential at $O_1$ or $O_2$ is

$$dV_{O_1 \text{ or } O_2} = \frac{\rho I}{4\pi^2} \cdot \left[ \frac{x \{x - (L \mp l)/2\} + y^2 + z^2}{(x^2 + y^2 + z^2)^{3/2}} \right]$$

$$+ \frac{4L^3}{(L^2 - l^2)^2} \cdot \frac{(L^2 - l^2)}{(x - L/2)^2} \left[ \frac{x - (L \mp l)/2}{(x^2 + y^2 + z^2)^{3/2}} \right]$$

$$+ \frac{(x - L)}{(x - L)^2} \left[ \frac{x - (L \mp l)/2}{(x^2 + y^2 + z^2)^{3/2}} \right]$$

$$+ \frac{1}{(x - L)^2 + y^2 + z^2}^{3/2} \left[ \frac{x - (L \mp l)/2}{(x^2 + y^2 + z^2)^{3/2}} \right]$$

$$+ \frac{1}{(x - L)^2 + y^2 + z^2}^{3/2} \left[ \frac{x - (L \mp l)/2}{(x^2 + y^2 + z^2)^{3/2}} \right].$$  \hspace{1cm} (16)

Integrating expression (16) over $x$ and $y$ from minus to plus infinity and dividing by the normalizing factor $\pi \left( \frac{2L(L^2 + l^2)}{l} \right)$, we get

$$\text{DIC(N)}_{S \text{, laterallog}} = \frac{\rho I}{2\pi} \frac{2L(L^2 + l^2)}{l} \cdot \left[ \frac{1}{(L^2 - l^2)^{1/2}} \right] \left[ \frac{1}{(L^2 + l^2)^{3/2}} \right].$$  \hspace{1cm} (17)

Fig. 3. Geometry of surface laterolog system.
Homogeneous ground: dipolar systems

With the plan geometry of Figure 4 and origin at the midpoint of current dipole in this case, the coordinates of the current and potential electrodes are

\[+I: (-l/2, 0, 0)\]
\[-I: (l/2, 0, 0)\]
\[P_1: [(L \cos \theta_1 - l \cos \theta_2/2), (L \sin \theta_1 - l \sin \theta_2/2), 0]\]
\[= (A, B, 0)\]
\[P_2: [(L \cos \theta_1 + l \cos \theta_2/2), (L \sin \theta_1 + l \sin \theta_2/2), 0]\]
\[= (C, D, 0)\].

The potential \(V_D\) at any point \((x, y, z)\) in the ground due to \(+I\) and \(-I\) is

\[V_D(x, y, z) = \frac{\rho I}{2\pi} \frac{1}{\sqrt{(l/2 + A)^2 + B^2}^{1/2}}\]

Integrating both sides of equation (19) over \(x\) and \(y\) and dividing by the normalizing factor

\[\frac{\rho I}{2\pi} \frac{1}{\sqrt{(l/2 + A)^2 + B^2}^{1/2}}\]

the potential difference between \(P_1\) and \(P_2\) caused by the polarized volume element is one obtains

\[dV_{P_1P_2}^D = \frac{\rho I}{4\pi^2} \int dz \left[\frac{(x - A)(x + l/2) + y(y - B) + z^2}{\{(x - A)^2 + (y - B)^2 + z^2\}^{3/2}\{(x + l/2)^2 + y^2 + z^2\}^{3/2}} - \frac{(x - C)(x + l/2) + y(y - D) + z^2}{\{(x - C)^2 + (y - D)^2 + z^2\}^{3/2}\{(x + l/2)^2 + y^2 + z^2\}^{3/2}} + \frac{(x - C)(x - l/2) + y(y - D) + z^2}{\{(x - C)^2 + (y - D)^2 + z^2\}^{3/2}\{(x - l/2)^2 + y^2 + z^2\}^{3/2}} - \frac{(x - A)(x - l/2) + y(y - B) + z^2}{\{(x - A)^2 + (y - B)^2 + z^2\}^{3/2}\{(x - l/2)^2 + y^2 + z^2\}^{3/2}}\right] dx dy.\]
\[ V_1 = \frac{\rho_1 I}{2\pi} \left[ \frac{1}{(x^2 + y^2 + z^2)^{3/2}} + \sum_{n=1}^{\infty} \frac{k^n}{\left\{ x^2 + y^2 + (z + 2nh)^2 \right\}^{1/2}} \right] \] (22)

and

\[ V_2 = \frac{\rho_1 I}{2\pi} (1 + k) \left[ \frac{1}{(x^2 + y^2 + z^2)^{3/2}} + \sum_{n=1}^{\infty} \frac{k^n}{\left\{ x^2 + y^2 + (z + 2nh)^2 \right\}^{1/2}} \right] \] (23)

For a parallel dipole array, for instance, \( \theta_z = 0 \).

With \( \theta_z = \pi/4, l = 0.1, \) and \( L = 1.0, \)

\[ A = 0.657, \quad B = 0.707, \]
\[ C = 0.757, \quad D = 0.707, \]
\[ M = -0.005, \]

and

\[ \text{DIC(N) dipole} = \frac{4\pi}{M} \left[ \frac{1}{(A - l/2)^2 + B^2 + 4z^2}^{3/2} - \frac{1}{(A - l/2)^2 + B^2 + 4z^2} + \frac{1}{(C + l/2)^2 + D^2 + 4z^2}^{3/2} + \frac{1}{(C + l/2)^2 + D^2 + 4z^2} \right]. \] (20)

Inhomogeneous ground: two-layer model with Wenner, Schlumberger, modified unipole, and two-electrode arrays

Though our investigations relate primarily to homogeneous ground, the method developed above is applicable to inhomogeneous ground as well. Provided that a theoretical solution to the boundary value problem exists, one can use this method to compute the contribution to the total signal by any desired portion of the ground—inhomogeneous, inhomogeneous, isotropic, or anisotropic.

Let us examine the depth investigation characteristics for the Wenner, Schlumberger, modified unipole, and the two-electrode systems in the simple case of a two-layered half-space, where a layer of thickness \( h \) and resistivity \( \rho_1 \) rests over a half-space of resistivity \( \rho_2 \). For the current electrode +I at \( (0, 0, 0) \), the potentials \( V_1 \) and \( V_2 \) at any point \( (x, y, z) \) for \( 0 \leq z \leq h \) and \( z \geq h \) are

\[ V_1 = \frac{\rho_1 I}{2\pi} \left[ \frac{1}{(x^2 + y^2 + z^2)^{3/2}} + \sum_{n=1}^{\infty} \frac{k^n}{\left\{ x^2 + y^2 + (z + 2nh)^2 \right\}^{1/2}} \right] \] (24)

from the results (A13) and (A14), and, for \( z \geq h \),

\[ \text{DIC(Wenner)} = \left( 1 + k \right) \left[ \text{right hand side of relation (24)} \right]. \] (25)

The normalizing factor for the Wenner system, derived directly or found by integrating equations (24) and (25) between proper limits and adding, is

\[ \text{NFWenner} = \frac{\rho_1 I}{2\pi} \left[ \frac{3}{L} + 4 \sum_{n=1}^{\infty} \frac{k^n}{\left\{ (L^2/9 + 4n^2h^2)^{1/2} \right\}} \right]. \] (26)
Similar expressions can be derived for the other three systems.

RESULTS AND DISCUSSION

It will have been noted that all our formulas are expressed in terms of a common yardstick of distance $L$, the separation between the two outermost electrodes in any system. The electrodes at infinity, where they exist, do not come into the picture, as they have no effect on the measured quantities. This choice seems natural and reasonable to us from both theoretical and the practical points of view, and allows a common reference for comparing the depths of investigation or depth investigation characteristics of the various electrode systems. For dipolar arrangements, $L$ represents the distance between the centers of the source and receiver dipoles, since the lengths $l$ of the dipoles themselves are supposed to be negligibly small. We have taken $l=0.1L$ and ignored the fact that the distance between the two extreme electrodes is not quite $L$, but somewhat larger. In cases where $l$ is not negligible, $L$ will no longer stand for the distance between the centers and one will need to use the actual distance between the two farthest electrodes.

With $L=ds=1.0$, formulas (10), (11), (12), (13), (17), and (20) have been used to compute the curves plotted in Figure 5 a and b. The curves begin at zero for zero depth, rise to a maximum, and then fall off to zero again at large depths. The parallel and polar dipole arrangements (curves 1 and 2 in Figure 5b) exhibit a secondary negative peak of much smaller magnitude before finally approaching zero at greater depths. The depths of investigation, according to our definition, are approximately

1. Two electrode
2. Equatorial or azimuthal dipole ($\theta_i=\pi/4$) 0.25

![Diagram](image)

**Fig. 5.** Depth investigation characteristics: (a) Wenner, Schlumberger, surface laterolog, modified unipole and two electrode arrays; (b) Some dipole arrangements including polar and equatorial arrays.
3. Perpendicular dipole ($\theta_1=\pi/4$) 0.20
4. Polar or radial dipole ($\theta_1=\pi/4$) 0.195
5. Parallel dipole ($\theta_1=\pi/4$) 0.18
6. Modified unipole 0.18
7. Surface laterolog ($l=0.1L$) 0.17
8. Surface laterolog ($l=0.2L$) 0.135
9. Schlumberger 0.125
10. Wenner 0.11.

The simplest two-electrode arrangement has the largest depth of investigation—more than three times that of the Wenner array, whose depth of investigation is the least, or more than twice that of the focused surface laterolog. This means that, in order to get the same information in resistivity sounding, the maximum separation with the two-electrode system needs to be only about one-third that of a Wenner or Schlumberger array (see discussion on vertical resolution later).

If $L$ is made 0.5 instead of unity in the two-electrode arrangement, the array's depth investigation characteristic curve becomes identical with that of the modified unipole; that is, curves 4 and 5 in Figure 5a coincide. Even then, the depth of investigation for the two-electrode system is a good deal greater than those for Wenner or Schlumberger arrays and somewhat larger than that for the surface laterolog.

The depth of investigation for the equatorial dipole array is exactly double that for the Schlumberger array. If the value of $L$ in the equatorial dipole array were taken as 0.5 instead of unity, the equatorial dipole's depth investigation characteristic—curve 4 in Figure 5b—would have coincided with that for the Schlumberger array—curve 2 in Figure 5a. This equivalence is well known.

For the surface laterolog array, as $l$ becomes smaller, the depth investigation characteristic approaches that of the modified unipole array. This is understandable. $I_0 \to 0$ as $l \to 0$, so that in the limit the two systems are one and the same. For larger values of $l$, the depth of investigation of the surface laterolog falls.

A very important fact emerges from Figure 5: as far as depth of investigation is concerned, a focused system forcing more current toward the target is not necessarily superior to an unfocused arrangement. One has only to compare the curve for the ideally nonfocused two-electrode arrangement with that for the surface laterolog or the modified unipole array. Let it also be noted that the modified unipole array has a maximum current density at $z=0.355L$ (Gupta and Bhattacharya, 1963), but this maximum apparently has no connection with the system's depth of investigation.

The contributions from all the polarized ground elements ($dxdydz$) are not of the same sign, as can be appreciated by visualizing the current lines. The subsidiary negative peaks in two of the depth investigation characteristics indicate those depths and systems for which the negative contributions predominate. As in electromagnetics (Roy and Dhar, 1970), contributions from portions of ground can cancel those from other portions.

The fact that, for small depths, the curves have zero or near-zero ordinates might tempt one to conclude that all the electrode systems are insensitive to the material very close to the ground surface. Unfortunately, this is not true. The curves attain a zero ordinate at zero depth not because the contributions from the volume elements in the topmost or shallowest layer are individually zero. On the contrary, the contributions from the elements at the electrodes themselves reach extremely large magnitudes (singularities), although they cancel each other in the summation process in the $x$ and $y$ directions. If the ground happens to be inhomogeneous near the surface, the cancellation will be nowhere near perfect and large residuals will occur. Thus, the sensitivity of direct current electrode systems to irregularities in the neighborhood of the electrodes is always high, and nothing much can be done about it. This property of direct current systems is in sharp contrast with that of electromagnetic dipole systems, for some of which, the contributions from the volume elements along the line of electrodes can be individually zero.

The depths of investigation obtained in this paper are absolute, while those discussed in the introduction, except for the treatment by Evjen, are relative or comparative. It is satisfying to note that Evjen's conclusion about the depth of investigation of the Wenner system checks exactly with ours. It is not so satisfying, however, that our results differ substantially from those of Al'pin in relations (3); although one must remember that the ground configurations are different in the two treatments. According to Al'pin, for instance, the radial dipole has the same depth of investigation as that for the Schlumberger array.

<table>
<thead>
<tr>
<th>Polarized Ground Elements</th>
<th>Depth of Investigation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel dipole ($\theta_1=\pi/4$)</td>
<td>0.18</td>
</tr>
<tr>
<td>Polar or radial dipole ($\theta_1=\pi/4$)</td>
<td>0.195</td>
</tr>
<tr>
<td>Perpendicular dipole ($\theta_1=\pi/4$)</td>
<td>0.20</td>
</tr>
<tr>
<td>Modified unipole</td>
<td>0.18</td>
</tr>
<tr>
<td>Surface laterolog ($l=0.1L$)</td>
<td>0.17</td>
</tr>
<tr>
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<td>0.135</td>
</tr>
<tr>
<td>Schlumberger</td>
<td>0.125</td>
</tr>
<tr>
<td>Wenner</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Our figures indicate that, spacing for spacing, the radial dipole array with $\theta_1 = 45$ degrees is approximately 1.5 times ($= 0.195/0.125$) superior to the Schlumberger arrangement. For the parallel dipole array, for another instance, the depth of investigation according to relation (3) becomes zero (!) for $\theta = \theta_1 = 45$ degrees, whereas our value is 0.18$L$. In other words, Al'pin's formula would indicate that, for a parallel dipole with $\theta = 45$ degrees, no finite spacing, however large, will yield any information about the ground! On the other hand, for $\theta = 54^\circ 44' 8''$, the parallel dipole would have an infinitely large depth of investigation, and should therefore see deep targets even with very small spacing! Physically, these results are not easy to comprehend, although Keller (1966) has suggested an explanation for the second case. For the perpendicular dipole, as another example, the depth of investigation obtained by Al'pin is 1.33 times that for the Schlumberger array. Our value again is 1.5, as for radial dipole. The findings of Keller and Frischknecht (1966), Fröhlich (1967), and Keller (1966), summarized in the introduction, seem to be in general agreement with our results.

The high depth of investigation of the two-electrode system is counterbalanced somewhat by its low vertical resolution in case of more than one target, with one below another. If the inverse of the width of each curve at its half-maximum points is taken as an indication, the different electrode systems arrange themselves as follows in order of decreasing vertical resolution:

1. Wenner
2. Schlumberger
3. Parallel dipole $\theta_1 = \pi/4$
4. Polar or radial ($\theta_1 = \pi/4$) dipole
5. Perpendicular dipole ($\theta_1 = \pi/4$)
6. Surface laterolog $l = 0.2L$
7. Surface laterolog with $l = 0.1L$ and modified unipole
8. Equatorial or azimuthal ($\theta_1 = \pi/4$) dipole
9. Two electrode

If it were not for the subsidiary negative peaks, the above order would also have been the order of the magnitudes of the peaks. Again, focused systems do not carry any special significance or advantage.

If one has to use a focused arrangement, a modified unipole is the obvious choice, because it uses only three electrodes as against the seven of the laterolog system and yet has the same general performance as the latter. This is a qualified remark and will hold only if the horizontal resolution—a property not investigated in this paper—of the laterolog does not turn out to be vastly superior to that of the modified unipole system.

The total signal measured on the ground surface is given in each case by the area under the curve concerned, multiplied by the normalization factor. As they are drawn in Figure 5, the areas under all the curves are the same, but do not equal unity, since $dz = 1$ in our computation.

Figure 6 displays some theoretical resistivity

![Figure 6](https://example.com/figure6.png)

**Fig. 6.** Resistivity sounding on a two-layer model with $h = 1$, $k = (\rho_2 - \rho_1)/(\rho_2 + \rho_1) = 0.3, 0.6, 0.9$ (or $\rho_2/\rho_1 = 1.86, 4, 19$) for two electrode, polar dipole, Schlumberger, and Wenner arrays.
 sounding curves over a two-layer model with \( h \) = thickness of top layer = unity and \( k \) = reflection factor = 0.9, 0.6, and 0.3 (or \( \rho_2/\rho_1 = 19, 4, \) and 1.86) for the two-electrode, polar dipole, Schlumberger, and Wenner electrode arrangements. The curves verify that the two-electrode system has a depth of investigation much larger than those of the other three; and that among the latter, the depth of investigation falls from polar dipole to Schlumberger to Wenner.

Figures 7 and 8 show the depth investigation characteristics with Wenner, Schlumberger, modified unipole, and two-electrode systems for two-layer models with \( L = 1, k = +0.5 \) and \(-0.5\), and \( h = 0.1, 0.2, \) and 0.3 [vide formulas (24) and (25), for instance, for the Wenner array]. Due to the terms involving \( 2a+2nh \), the ordinates at zero depth are no longer zero, but have positive or negative values according to the sign of \( k \). In order to avoid crowding in Figure 8, the negative part of the curves—almost straight lines—are not shown, except where they meet the ordinate. It may be seen that, for \( h = 0.3 \), only in the two-electrode system does a substantial portion of the signal come from the lower layer.

Figure 9 shows a few model tank resistivity profiles across a vertical, finitely conducting vein for the Wenner, modified unipole (which is the same as surface laterolog), and the two-electrode arrangements. Even with an \( L \) that is half of those of the other two, the two-electrode profiles are distinctly superior both in regard to shape and magnitude.

**CONCLUDING REMARKS**

The treatment in this paper is simple and relates chiefly to homogeneous ground. The results, however, do appear remarkably fundamental both in regard to principles and practice, and should qualitatively guide a field geophysicist in
his choice of an electrode system and its spacing for any real inhomogeneous problem. As stated and illustrated already, there is no difficulty in extending the method to inhomogeneous media, although such an extension would be unlikely to yield further results of comparable interest.

We would like to emphasize again that the numerical fractions for depths of investigation are not factors that can be used to predict depths of targets from positions of maximum, minimum, inflection point, etc. It is also worth repeating that the depth of investigation, as defined in this

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**FIG. 8.** Depth investigation characteristic in a two-layer model with \( L = 1, k = -0.5 \), and \( h = 0.1, 0.2, \) and \( 0.3 \). Note that the ordinates for small depths are negative.
ACKNOWLEDGMENT

We are thankful to Mr. D. M. K. Gupta for computation, to Mr. P. Krishnaswamy for diagram tracing, and to Mr. R. Acharya for photocopies of the diagrams.

REFERENCES


Doll, H. G., 1949, Introduction to induction logging and application to logging of wells drilled with oil-base
The integral in (A1) can be split up into three additive parts $I_1$, $I_2$, and $I_3$. Let us take the Fourier transform of $I_1$ with respect to $a$ and $b$, substitute $(a - x) = \alpha$ and $(b - y) = \beta$, and rearrange terms. With $i = \sqrt{-1}$ and $u$ and $v$ as the angular spatial frequencies in the $a = x$ and $b = y$ directions, we then have

$$I_1(u, v, c, z) = -\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \alpha \cdot e^{-iux_1} \cdot e^{i\beta y} d\alpha d\beta \cdot \left(\frac{x \cdot e^{-iux_1} \cdot e^{-iuy}}{(x^2 + y^2 + z^2)^{3/2}}\right)$$

$$= (2\pi)^2 \cdot u^2 \cdot \frac{e^{-(2c-\alpha)^2} u^2 v^2}{u^2 + v^2}, \quad \text{for } (z - c) > 0$$

or

$$I_3(u, v, c, z) = (2\pi)^2 \cdot v^2 \cdot \frac{e^{-(2c - \beta)^2} u^2 v^2}{u^2 + v^2}, \quad \text{for } (z - c) > 0$$

throughout this paper, integrals of the general type

$$I = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{x(x - a) + y(y - b) + z(z - c)}{(x^2 + y^2 + z^2)^{3/2}} \frac{dxdy}{(x - a)^2 + (y - b)^2 + (z - c)^2}^{3/2}$$

have been encountered. The integration can be carried out analytically by using the following three Fourier transform pairs:

$$\frac{1}{(x^2 + y^2 + z^2)^{1/2}} \leftrightarrow 2\pi \cdot \frac{e^{-1z_1} u^2 v^2}{\sqrt{u^2 + v^2}}.$$  \hspace{1cm} (A2)

$$\frac{|z|}{(x^2 + y^2 + z^2)^{1/2}} \leftrightarrow 2\pi \cdot \frac{e^{-1z_1} u^2 v^2}{\sqrt{u^2 + v^2}}.$$  \hspace{1cm} (A3)

and

$$\frac{1}{(x^2 + y^2 + z^2)^{1/2}} \leftrightarrow 2\pi \cdot \frac{e^{-1z_1} u^2 v^2}{\sqrt{u^2 + v^2}}.$$  \hspace{1cm} (A4)

APPENDIX

Throughout this paper, integrals of the general type
\[ I_3(u, v, c, z) = (2\pi)^2 \cdot e^{-(2z-c)^2 \sqrt{v^2 + z^2}}, \]

for \((z - c) > 0\) \(\text{(A9)}\)

or \(- (2\pi)^2 \cdot e^{-(2z-c)^2 \sqrt{v^2 + z^2}}, \)

for \((c - z) > 0\), \(\text{(A10)}\)

from transform pair \((A4)\). Therefore,

\[ I(u, v, c, z) = I_1(u, v, c, z) + I_2(u, v, c, z) \]

\[ + I_3(u, v, c, z) \]

\[ = (2\pi)^2 \cdot 2 \cdot e^{-(2z-c)^2 \sqrt{v^2 + z^2}}, \]

for \((z - c) > 0\), \(\text{(A11)}\)

or 0, for \((c - z) > 0\), \(\text{(A12)}\)

from equations \((A5)\), \((A6)\), \((A7)\), \((A8)\), \((A9)\), and \((A10)\). Extracting the inverse transform of \(I(u, v, c, z)\) by using the transform pair \((A4)\) yields

\[ I(a, b, c, z) = \frac{4\pi(2z-c)}{\{a^2 + b^2 + (2z-c)^2\}^{3/2}} \]

for \((z - c) > 0\), \(\text{(A13)}\)

\[ = 0 \text{ for } (c - z) > 0. \] \(\text{(A14)}\)

It is intriguing that

\[ I_1(a, b, c, z) + I_2(a, b, c, z) = \pm I_3(a, b, c, z), \]

for \((z - c) \geq 0\),

which means that, for any horizontal layer, the total effect of the horizontal dipoles at the measuring point(s) is equal numerically to that of the vertical dipoles.