

THE APPLICATION OF LINEAR FILTER THEORY TO THE DIRECT INTERPRETATION OF GEOELECTRICAL RESISTIVITY SOUNDING MEASUREMENTS*

BY

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ABSTRACT

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Koefoed has given practical procedures of obtaining the layer parameters directly from the apparent resistivity sounding measurements by using the raised kernel function $H(\lambda)$ as the intermediate step. However, it is felt that the first step of his method—namely the derivation of the H curve from the apparent resistivity curve—is relatively lengthy.

In this paper a method is proposed of determining the resistivity transform $T(\lambda)$, a function directly related to $H(\lambda)$, from the resistivity field curve. It is shown that the apparent resistivity and the resistivity transform functions are linearly related to each other such that the principle of linear electric filter theory could be applied to obtain the latter from the former. Separate sets of filter coefficients have been worked out for the Schlumberger and the Wenner form of field procedures. The practical process of deriving the T curve simply amounts to running a weighted average of the sampled apparent resistivity field data with the pre-determined coefficients. The whole process could be graphically performed within an quarter of an hour with an accuracy of about 2%.

INTRODUCTION

Direct interpretation of geoelectrical sounding resistivity measurements is carried out in two steps. In the first step the kernel function $B(\lambda)$ (see eq. (1)) or some modified version of this function is evaluated from the field measurements, and in the second step the layer parameters (resistivities ρ_i and thicknesses d_i) are derived from it.

The logic behind this approach is that the kernel function is dependent only on the layer parameters, and an expression relating it to the field measurements can be obtained by mathematical processes.

These suggestions (Slichter 1933, Pekeris 1940)—although made about four decades ago—found relatively little application, mainly because of the non-availability of a simple procedure to compute the kernel. Consequently inter-

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preters resorted to indirect techniques like the comparison of observed apparent resistivity (ρ_a) curves with precalculated curves for known earth models.

Almost a quarter of a century elapsed before interest in direct interpretational methods was revived. The credit goes to Koefoed (1968) who gave practical procedures to directly interpret the apparent resistivity field curves, along the lines of interpretation suggested earlier. For the intermediate step, however, he chose to use the raised kernel function $H(\lambda)$ instead of $B(\lambda)$. He showed that relative variations in the apparent resistivity were not of the same order of magnitude in the corresponding kernel curve, and thus any method based on the determination of this function as the intermediary step would lead to loss of information and hence to incorrect interpretation. Koefoed (1970) subsequently modified the second step of his method by introducing standard graphs to accelerate the derivation of the layer distribution from the resistivity transform $T(\lambda)$, a function directly related to $H(\lambda)$.

It is, however, felt that Koefoed's first step is relatively lengthy which could be of disadvantage to the application of direct methods in resistivity interpretation. In this paper therefore an attempt is made to devise a simple and quick procedure to obtain the resistivity transform from the apparent resistivity field curve. An analysis of the properties of the T function given below demonstrates the suitability of this function as the intermediary step in direct interpretation:

1. It is solely determined by the layer distribution.
2. It is an unambiguous representation of the ρ_a function.
3. For small and large values of τ/λ it approaches the ρ_a curve.

It will be shown that the ρ_a and T functions are linearly related to each other such that the principle of electric filter theory could be applied to derive the T curve from the ρ_a curve.

Swartz and Sokoloff (1954), Dean (1958), Robinson and Treitel (1964) have all demonstrated the applicability of electric filter theory to solve physical problems which are linear in nature. One essential difference that might be stressed here is that electric or "real" time filters depend on the excitation of energy at $t = 0$ to produce an output and thus can have only finite responses for $t \geq 0$, whereas in filters used for data processing or to handle physical problems there can be responses of the filter for both positive and negative values of the independent variable. The implication of this is that in electric filters the output depends only on the present and past values of the input, whereas in the latter case the output in addition depends also on the future values of the input (Dean 1958, Robinson and Treitel 1964).

THE FUNDAMENTAL RELATION BETWEEN THE POTENTIAL, THE RESISTIVITY TRANSFORM, AND THE APPARENT RESISTIVITY OF A STRATIFIED EARTH

The relations can be derived starting from the expression for the potential (Stefanescu 1930) due to a point source of current I at a point $(r, 0)$ on the surface of a stratified earth:

$$\begin{aligned} V(r, 0) &= \frac{\rho_1 I}{2\pi} \left[\frac{1}{r} + 2 \int_0^{\infty} B(\lambda, k, d) J_0(\lambda r) d\lambda \right] \quad (1) \\ &= \frac{\rho_1 I}{2\pi} \left[\int_0^{\infty} J_0(\lambda r) d\lambda + 2 \int_0^{\infty} B(\lambda, k, d) J_0(\lambda r) d\lambda \right] \\ &= \frac{I}{2\pi} \int_0^{\infty} T(\lambda) J_0(\lambda r) d\lambda, \quad (1a) \end{aligned}$$

where

$$T(\lambda) = \rho_1 [1 + 2B(\lambda)]. \quad (2)$$

The following relation exists

$$H(\lambda) = B(\lambda) + 1/2 = T(\lambda)/(2\rho_1),$$

where

- λ = integration variable; has the dimension of inverse distance,
- $B(\lambda)$ = kernel function,
- $T(\lambda)$ = resistivity transform function,
- $H(\lambda)$ = raised kernel function.

Apparent Resistivities

In the Schlumberger arrangement we have

$$\rho_{aS} = -\frac{2\pi}{I} r^2 \left(\frac{dV}{dr} \right)_{r=s} \quad (3)$$

where s is half the current electrode spacing. The subscript S and similarly W below in the symbols of the apparent resistivity and other quantities signify the type of arrangement to which the quantities refer.

Obtaining the potential gradient from (1a) and substituting in (3) we have

$$\rho_{aS} = s^2 \int_0^{\infty} T(\lambda) J_1(\lambda s) \lambda d\lambda. \quad (4)$$

In the Wenner arrangement we have

$$\rho_{aW} = \frac{2\pi a}{I} \Delta V_W, \quad (5)$$

where

a = electrode spacing, and
 $\Delta V_W = V_a - V_{2a}$ = Wenner potential difference.

Obtaining the potential difference from (1a) and substituting in (5) we have

$$\rho_{aW} = a \int_0^{\infty} T(\lambda) [J_0(\lambda a) - J_0(\lambda 2a)] d\lambda \quad (6)$$

THE LINEAR FILTER ANALOGY

An explicit expression for the resistivity transform can be obtained by applying Hankel's inversion (see Watson 1966) of the Fourier-Bessel integral to the expression of the apparent resistivity in the Schlumberger arrangement given by (4):

$$T(\lambda) = \int_0^{\infty} [\rho_{aS}(s) J_1(\lambda s)/s] ds \quad (7)$$

New variables are introduced defined by

$$x = \ln s \quad \text{and} \quad y = \ln (1/\lambda) \quad (8)$$

Substituting (8) in (7) we have

$$T(y) = \int_{-\infty}^{\infty} \rho_{aS}(x) J_1(1/e^{y-x}) dx \quad (9)$$

This is a convolution integral (see Lee 1960) relating the input $\rho_{aS}(x)$ to the output $T(y)$. There are, however, several other ways to prove that the ρ_a and T functions belong to a linear system, for example by the aid of the superposition theorem (see Anders 1964; Pickles 1967) applicable to linear systems. This can be shown directly starting from the expression of the apparent resistivity for both configurations.

For the Wenner arrangement a convolution integral of as simple a form as (9) is unobtainable; nevertheless the Wenner transformation is also linear. This can be shown as follows

If $\rho'_{aW} = a \int_0^{\infty} T'(\lambda) [J_0(\lambda a) - J_0(\lambda 2a)] d\lambda$ is one resistivity curve and $T'(\lambda)$ is the corresponding transform and

$\rho''_{aW} = a \int_0^{\infty} T''(\lambda) [J_0(\lambda a) - J_0(\lambda 2a)] d\lambda$, a second one, then the transform of the sum of the two curves is the sum of the individual transforms:

$$(\rho'_{aW} + \rho''_{aW}) = a \int_0^{\infty} (T'(\lambda) + T''(\lambda)) [J_0(\lambda a) - J_0(\lambda 2a)] d\lambda$$

In fact the idea of Koefoed (1968) of approximating the Schlumberger and Wenner apparent resistivity field curves by partial resistivity functions and then obtaining the total raised kernel function as a sum of the corresponding partial raised kernels would not have been possible if the condition of linearity was not met.

FREQUENCY CHARACTERISTIC OF THE RESISTIVITY FILTER

In the frequency domain the input-output relationship given by (9) (see Bracewell 1965, Robinson 1967, Anders 1964) takes the simple algebraic form

$$F(f) = G_S(f) H_S(f) \text{ or } H_S(f) = F(f)/G_S(f) \quad (10)$$

where

$$F(f) \leftrightarrow T(y); G_S(f) \leftrightarrow \rho_{as}(x)$$

$H_S(f)$ = Frequency characteristic of the resistivity filter, or simply the filter characteristic.

The symbol \leftrightarrow is used in this paper to denote a Fourier transform pair (see appendix IV).

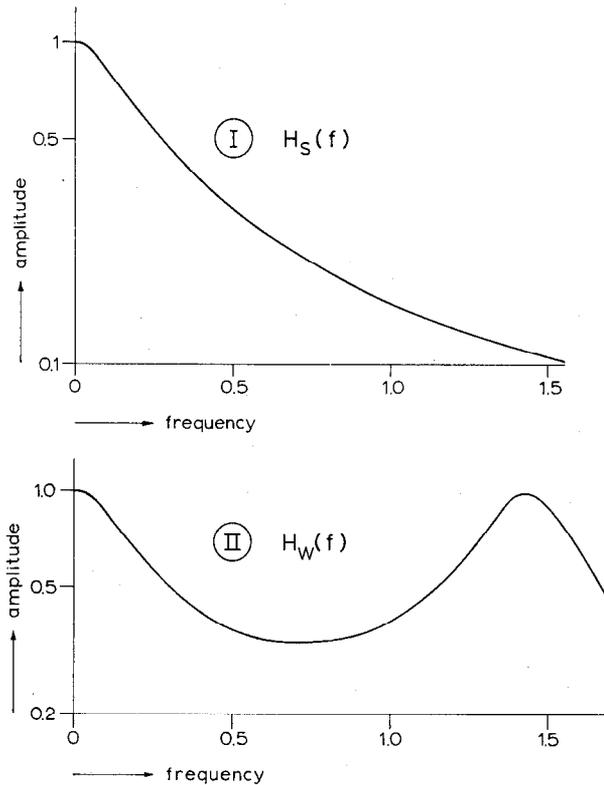


Fig. 1. Amplitude Fourier spectra of the resistivity filter characteristic for the (I) Schlumberger system and (II) Wenner system.

Since the transformation in the Wenner system is also linear the filter characteristic of that system is given by

$$H_w(f) = F(f) / G_w(f) \quad (11)$$

To determine the filter characteristic a partial resistivity function for the Schlumberger system (see appendix I) was chosen whose exact resistivity transform was known (Koefoed 1968). The Fourier transforms were evaluated numerically and their ratio yielded the filter characteristic. This was later verified by using the same approach to another set of functions. For the Wenner system, the Wenner partial resistivity function has to be used that can be readily obtained from the Schlumberger resistivity function (see appendix I).

The amplitude spectra of the filter characteristic for both configurations are presented in fig. 1. In both cases the amplitude is one at zero frequency. For the Schlumberger system the amplitude decreases to a constant value of 0.1, whereas for the Wenner system its behavior is different at higher frequencies. In both cases only the initial part of the amplitude spectra of the filter characteristic is needed.

PRINCIPLE OF THE PROPOSED METHOD

The apparent resistivity curve is sampled at equidistant points and replaced at the sample points by functions of the form of $\sin x/x$ called sinc functions. These functions have the property that they have the value of unity at the sample point where they are defined and zero at all other sample points. The resistivity curve is thus decomposed into a finite number of sinc functions, with the sample value as equivalent peak height and period determined by the sampling interval chosen. The sinc functions can now be treated individually as the input to the filter in lieu of the apparent resistivity function (in (9)), and the resistivity transform of each of these sinc functions can be obtained separately. Since the condition of linearity is valid, the sum of the resistivity transforms of the individual sinc functions yields the total resistivity transform of the whole apparent resistivity curve.

The above procedure is laborious and thus unsuitable. The digital approach is preferred which requires in the first instance determination of the response of the resistivity filter to a sinc function input. It will be referred to as the sinc response of the filter. Sampled values of this response give us the digital filter coefficients. The resistivity transform is obtained by running a weighted average of the sample values of the field curve with the filter coefficients. The basic problem thus is:

1. to determine the sampling interval,
2. to determine the coefficients.

DETERMINATION OF THE SAMPLING INTERVAL

Periodic sampling is applied because the theory of equispaced data is much simpler to use. The resistivity observations in the field correspond to geometrically increasing electrode separations which are not linear. However, switching over to the logarithmic scale makes the data linear and amenable to sampling.

In order that the apparent resistivity curve can be sampled effectively two conditions have to be satisfied

1. the ρ_a function must have a Fourier transform i.e. $\rho_a \leftrightarrow G(f)$
2. the amplitude spectrum of the Fourier transform must be band-limited i.e. $|G(f)| \approx 0$, for $f > f_c$ where f_c is the so-called cut-off frequency of the spectrum.

Moreover, care should be taken that the sampled values are truly representative of the sampled function, in other words, it should be possible, if so desired, to recover the function to a reasonable degree of accuracy from the sample values. This is only possible if the sampling obeys the following condition laid down in Shannon's theorem

$$0 \leq \Delta x \leq 1/(2f_c)$$

Thus the largest permissible sampling interval promising full recovery is

$$\Delta x = 1/(2f_c) \quad (12)$$

which is often referred to as the Nyquist-rule. In practice recovery is never complete because (i) most functions are not strictly band-limited (ii) for practical purposes only a finite number of sample points are considered against the theoretical requirement of sample points extending from minus to plus infinity.

To answer these questions it is necessary to perform a Fourier analysis of various apparent resistivity curves. As the expression of the apparent resistivity function is complicated the Fourier transform of the $T(y)$ functions were first determined and the Fourier transform of the apparent resistivity functions were then obtained from it by the application of (10). A total of about forty different resistivity distributions were studied which included two and three layer ascending, descending and bowl-shaped maximum and minimum types. It is hoped that these examples should yield sufficient knowledge about the general frequency behavior of resistivity curves. The Fourier amplitude spectra of $\rho_{a,S}(x)$ for a few cases are reproduced in figures 2 and 3.

The amplitude spectra show in general that the resistivity function is band-limited and hence sampling could be applied. For the two layer cases the amplitudes for lower frequencies approach to infinity, whereas for the three layer ones with $\rho_1 = \rho_2$ they have a singular point with infinite value at

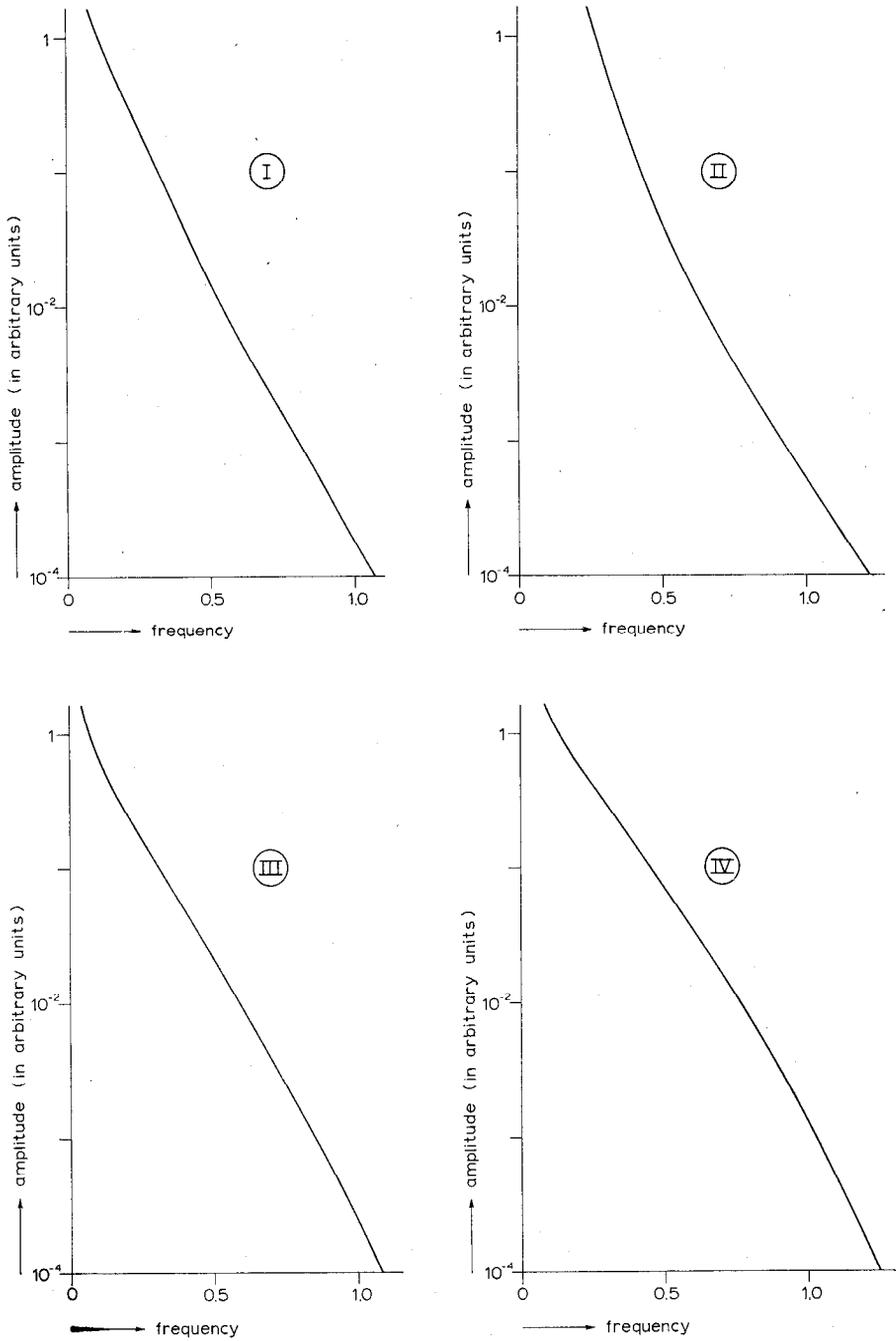


Fig. 2. Amplitude Fourier spectra of $\rho_{as}(x)$ for two layer cases with $d_1 = 1$, $\rho_1 = 1$, and (I) $k_1 = 0.3$ (II) $k_1 = 0.9$ (III) $k_1 = -0.3$ (IV) $k_1 = -0.9$.

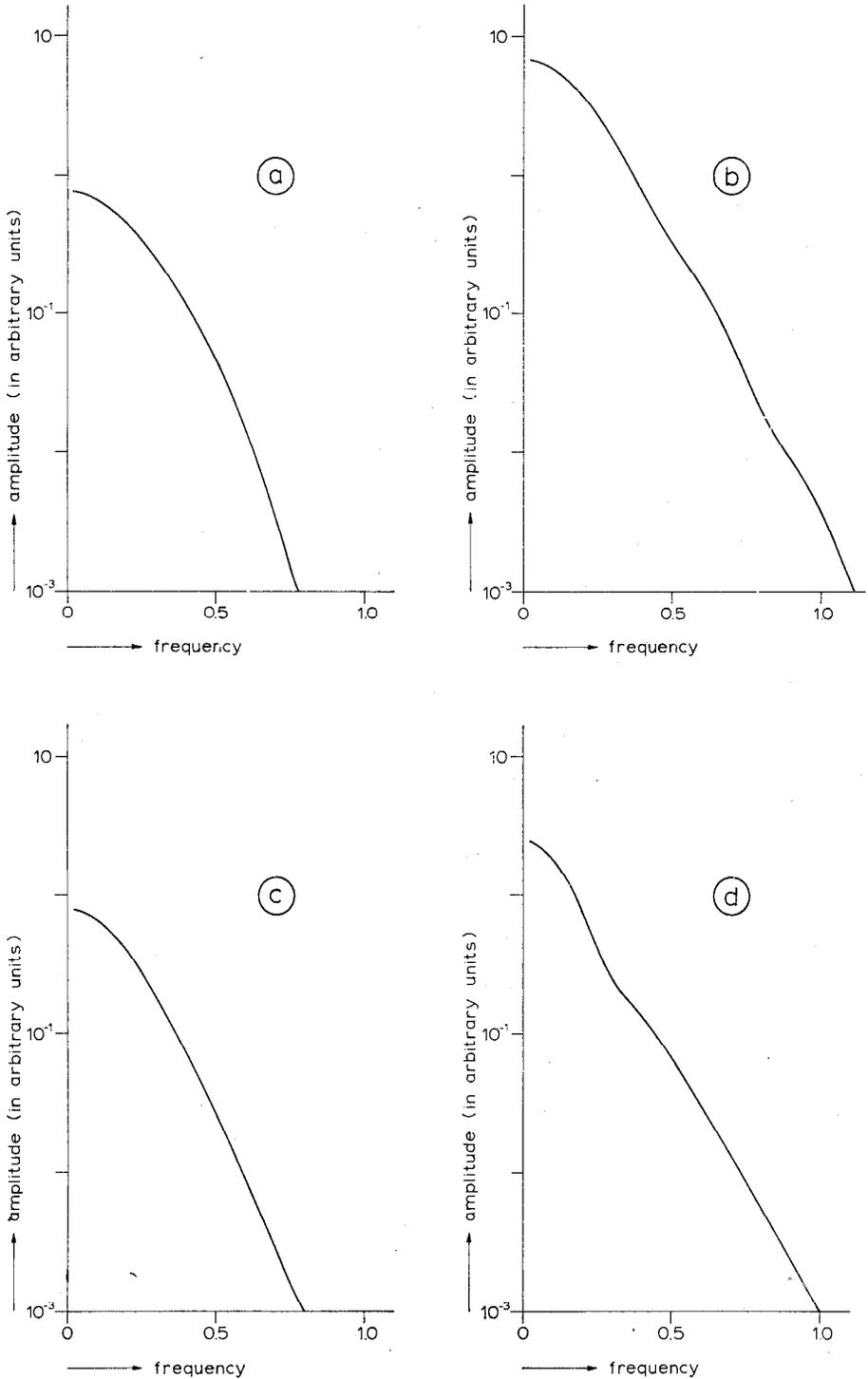


Fig. 3. Amplitude Fourier spectra of $\rho_{\alpha, \delta}(x)$ for three layer cases with $d_1 = 1, \rho_1 = \rho_3 = 1$, and (a) $d_2 = 2, k = 0.3$ (b) $d_2 = 5, k = 0.8$ (c) $d_2 = 3, k = -0.3$ (d) $d_2 = 3, k = -0.8$; where $k = k_1 = -k_2$.

$f = 0$. Although the pattern of decrease is quite encouraging from the sampling viewpoint, a unique choice of the cut-off frequency and hence according to (12) of the sampling interval is rather difficult, because of the uncertainty in the choice of the proper zero level. Therefore an indirect approach was applied. Three sampling intervals were chosen and used to reconstruct known two and three layer resistivity transform functions from their sample values (see appendices II and III). The T functions were selected for reconstruction as they have the general shape of resistivity curves and at the same time are far simpler to compute. The sampling intervals tried out are

$$\Delta x_a = \frac{1}{4} \ln 10; \Delta x_b = \frac{1}{3} \ln 10; \Delta x_c = \frac{1}{2} \ln 10.$$

The relative error in reconstruction of the function values at the intermediate points were also calculated and the largest deviations are shown in table 1 for the two layer cases and in table 2 for the three layer cases. It is immediately clear that both Δx_a and Δx_b could be used to sample the resistivity curves, in view of fact that the resistivity field data themselves are generally accurate to

TABLE I

Largest relative error (in %) between the original and reconstructed two layer transform function with $\rho_1 = 1$, $d_1 = 1$ and for various values of the reflection coefficient k_1 for different sampling intervals.

sampling interval	$\Delta x_a = \frac{1}{4} \ln 10$	$\Delta x_b = \frac{1}{3} \ln 10$	$\Delta x_c = \frac{1}{2} \ln 10$
$k_1 = 0.3$	$0.50.10^{-2}$	-0.04	-0.42
0.9	0.02	-0.11	-1.07
-0.3	$0.76.10^{-2}$	-0.07	0.57
-0.8	0.09	0.25	2.67
-0.9	0.19	0.64	6.24

TABLE II

Largest relative error (in %) between the original and reconstructed three layer transform function with $\rho_1 = \rho_3 = 1$, $d_1 = 1$, $d_2 = 3$ and for various values of k where $k = k_1 = -k_2$ for different sampling intervals.

sampling interval	$\Delta x_a = \frac{1}{4} \ln 10$	$\Delta x_b = \frac{1}{3} \ln 10$	$\Delta x_c = \frac{1}{2} \ln 10$
$k = 0.3$	$0.85.10^{-2}$	0.06	0.89
0.9	0.06	0.68	5.59
-0.3	$-0.59.10^{-2}$	0.06	0.67
-0.8	-0.03	0.24	0.96
-0.9	-0.04	0.28	1.73

only 3%. Hence our choice between the two was guided by the speed of application. Δx_a corresponds to four and Δx_b to three intervals in a factor 10 of the logarithmic paper on which the field resistivity curve appears. For illustration let us assume that a Schlumberger field survey is performed with the final current electrode spacing 600 m—i.e. $s = 300$ m. This means working with Δx_b in interpretation would save at least three sample points where the T values have to be obtained from the ρ_a sample values. Thus Δx_b is the recommended sampling interval. Putting the value of $\Delta x_b (\frac{1}{3} \ln 10 \approx 0.77)$ in (12) we get the value of 0.65 for the cut-off frequency.

SINC RESPONSE OF THE RESISTIVITY FILTER

The sinc response of the filter is obtained by treating the sinc function, with period determined by the sampling interval Δx_b , as the input to the filter. In the frequency domain it will be stated

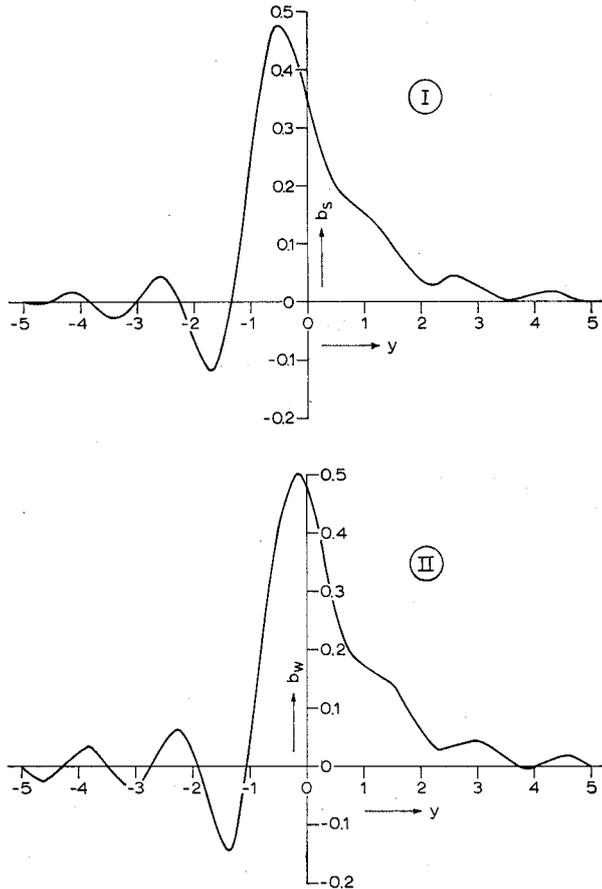


Fig. 4. Sinc response of the resistivity filter for the (I) Schlumberger system and (II) Wenner system.

for the Schlumberger system as:

$$B_S(f) = \xi(f) H_S(f), \text{ and} \quad (13)$$

for the Wenner system as:

$$B_W(f) = \xi(f) H_W(f) \quad (14)$$

where,

$\xi(f)$ = Fourier transform of the sinc function

$H(f)$ = Filter characteristic

$B(f)$ = Fourier transform of the sinc response.

It may be remarked here that $B(f)$ is zero for $f > f_c$, as the amplitude spectrum of the sinc function (the familiar block rectangular function) is zero beyond the cut-off frequency. This indicates that only the initial part of the filter characteristic (fig. 1) is utilized in the determination of $B(f)$ for both systems.

The sinc response $b(y)$ can now be recovered from $B(f)$ by the application of the inverse Fourier transform and is presented in fig. 4 for both systems.

DIGITAL FILTER COEFFICIENTS

The filter coefficients are the sampled values of the sinc response shown in fig. 4. The sampling interval has to be kept the same as for sampling the resistivity curves, i.e. $\frac{1}{3} \ln 10$, if it is desired to obtain the same form of output as the input. In digital operations it is possible to work with a finite number of coefficients; thus care should be taken that the sampled values do not occupy the crests and troughs of the response which would make the length of the filter virtually infinite.

Schlumberger system

The response in this configuration is favourable with respect to the sampling interval. Sampled values of it at a spacing of $\frac{1}{3} \ln 10$ constitute the twelve point filter shown in table III. It will be called the "long" filter for reasons which will become apparent in the subsequent discussion. This set is the true representation of the response and hence the truncation error—i.e. the error due to the use of only a finite interval of the response—is minimum. However, it has at least two practical disadvantages:

1. It has too large a number of coefficients which makes the operation slow and unhandy,
2. There are three coefficients for the response $y < 0$.

TABLE III
The "long" digital filter coefficients for the Schlumberger system.

a_{-3}	a_{-2}	a_{-1}	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
0.0060	-0.0783	0.3999	0.3492	0.1675	0.0858	0.0358	0.0198	0.0067	0.0051	0.0007	0.0018

TABLE IV
The "short" digital filter coefficients for the Schlumberger system.

a_{-2}	a_{-1}	a_0	a_1	a_2	a_3	a_4	a_5	a_6
-0.0723	0.3999	0.3492	0.1675	0.0858	0.0358	0.0198	0.0067	0.0076

The second point needs further clarification. It is well known in the theory of operation with digital filter coefficients that they have to be first reversed about the centre of the filtering operation i.e. about $y = 0$ and then applied to the input. This means that to yield the output corresponding to the last sample value on the observed curve (relating to the largest electrode spacing used) we still need three more sample values on which the coefficients a_{-1} , a_{-2} , a_{-3} can act upon. These three sample values are known as the future values of the input which were referred to before. It is obvious that in this problem they have to be obtained by extrapolating the observed curve to the right, and unless the extrapolation is reasonably accurate the output corresponding to the last section of the field curve might be affected. Extrapolation difficulties will arise when the asymptotic part of the resistivity curve is not reached by the field survey. It is also required to extrapolate upto 8 sample points to the left corresponding to the coefficients a_1 to a_8 to yield the output in the earlier part of the resistivity curve. But extrapolation to the left is no real problem as the apparent resistivity value approaches to ρ_1 for small electrode spacings.

Thus it is desired that the filter has a minimum number of coefficients corresponding to the response for $y < 0$, in order to reduce uncertainties in extrapolation to the right. An examination of table III gives us sufficient ground to reduce the twelve point filter to a nine point one, by accommodating the values of the coefficients a_{-3} , a_7 , a_8 into its neighbouring ones.

The resulting coefficients comprise the "short" filter shown in table IV. As a result of the shortening it is to be expected that the truncation error will be increased. In a later section (see fig. 11) it will be seen that the accuracy obtained with the short filter although lower than with the long one, is still reasonably good and satisfies our requirements. Moreover, it imparts speed to the application.

Wenner system

The sinc response, however, is not favourable in relation to the sampling interval of $\frac{1}{3} \ln 10$. Thus to avoid the unfortunate situation of having to work with a large number of filter points, the filter coefficients are so chosen that a_0 refers to the response value at abscissa value of $y = -\ln 1.616 = -0.48$ and not at $y = 0$. All other coefficients are then defined with respect to a_0 maintaining the constant spacing of $\frac{1}{3} \ln 10$. The implication of this is that the outputs (the transform values) will be shifted to the left by a factor of $\ln 1.616$ in relation to the input resistivity sample values. The nine point filter for the Wenner arrangement is shown in table V.

TABLE V
The digital filter coefficients for the Wenner system.

a_{-2}	a_{-1}	a_0	a_1	a_2	a_3	a_4	a_5	a_6
0.0212	-0.1199	0.4226	0.3553	0.1664	0.0873	0.0345	0.0208	0.0118

PRACTICAL PROCEDURE OF OBTAINING THE RESISTIVITY TRANSFORM

I. Numerical calculation by convolution

The convolution of the apparent resistivity sample values (R_m) with the filter coefficients (a_j) yields the transform values (T_m) at the sample point for the Schlumberger arrangement. This is stated in the following digitalized convolution statement

$$T_m = \sum_{j=-2}^6 a_j R_{m-j} \quad (15)$$

$$\text{for } m = 0, 1, 2, 3, 4, 5, \dots$$

Let us consider a simple example in which we say that the resistivity curve is defined at the sample points 0, 1, 2, 3, 4 and we like to determine the transform value at sample point 3. Putting $m = 3$ in (15) we have

$$T_3 = a_{-2}R_5 + a_{-1}R_4 + a_0R_3 + a_1R_2 + a_2R_1 + a_3R_0 + a_4R_{-1} + a_5R_{-2} + a_6R_{-3} \quad (16)$$

In (16) the values R_{-1}, R_{-2}, R_{-3} have to be obtained by extrapolating the resistivity curve to the left; R_5 is obtained by extrapolation to the right.

For the Wenner arrangement (15) could still be used if it is kept in mind that the transform values obtained refer to abscissa points $\ln 1.616$ to the left of the sample points.

2. Graphical process of application by superposition

We shall concentrate on this mode of application because of its importance to the field geophysicists. The same results as before can be obtained by summing up the responses after each input value has been operated on by the coefficients. The procedure given below refers only to the Schlumberger arrangement.

1. The filter coefficients from table IV are plotted on a sheet of double logarithmic paper, preferably of modulus of 62.5; a_{-2} is denoted by a dash as its value is negative and its contribution has to be subtracted. A cross is placed at ordinate value of 1 and $y = 0$. These details are illustrated in fig. 5.

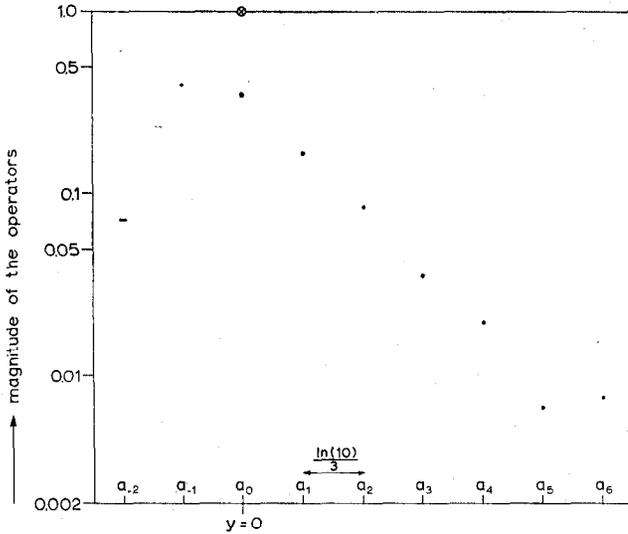


Fig. 5. The "short" digital filter coefficients for the Schlumberger system.

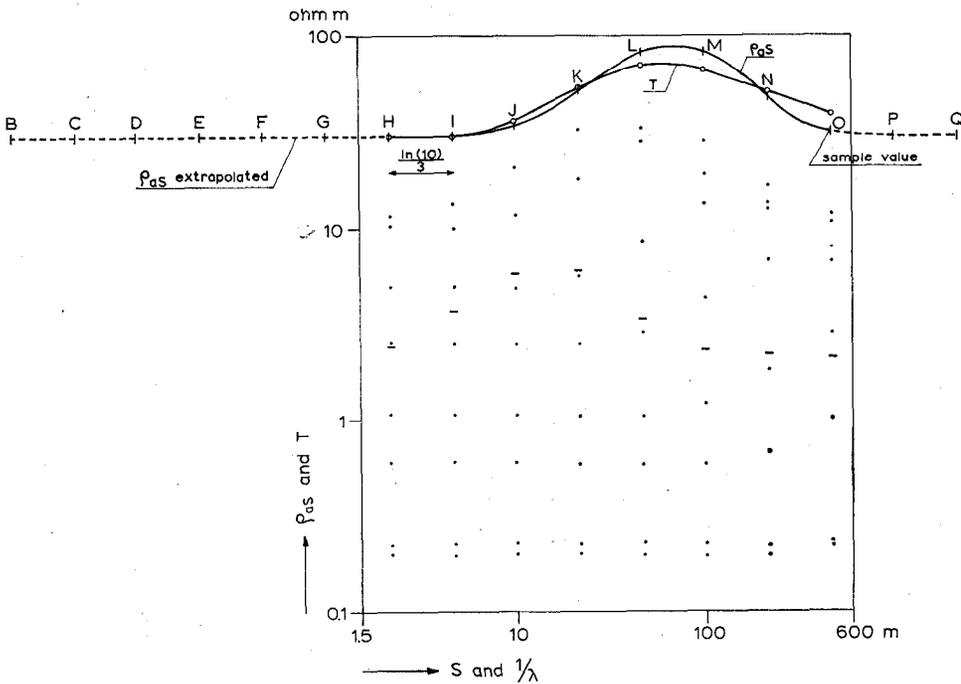


Fig. 6. Illustration of the graphical process of application of the method to a three layer Schlumberger apparent resistivity curve with $\rho_1 = \rho_3$.

2. The field resistivity curve is retraced on the top right portion of a transparent sheet of double logarithmic paper of the same modulus.

3. The sample values are marked by a dash on the resistivity curve at a constant spacing of $\frac{1}{3} \ln 10$. In fig. 6 they are defined from H to O . The resistivity curve is extrapolated to the left to yield six values (G to B) corresponding

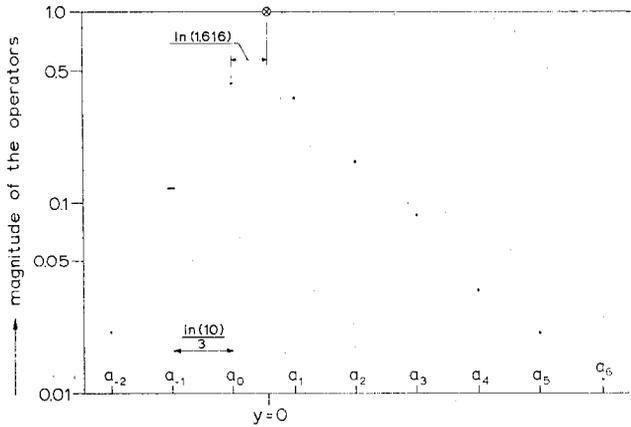


Fig. 7. The digital filter coefficients for the Wenner system.

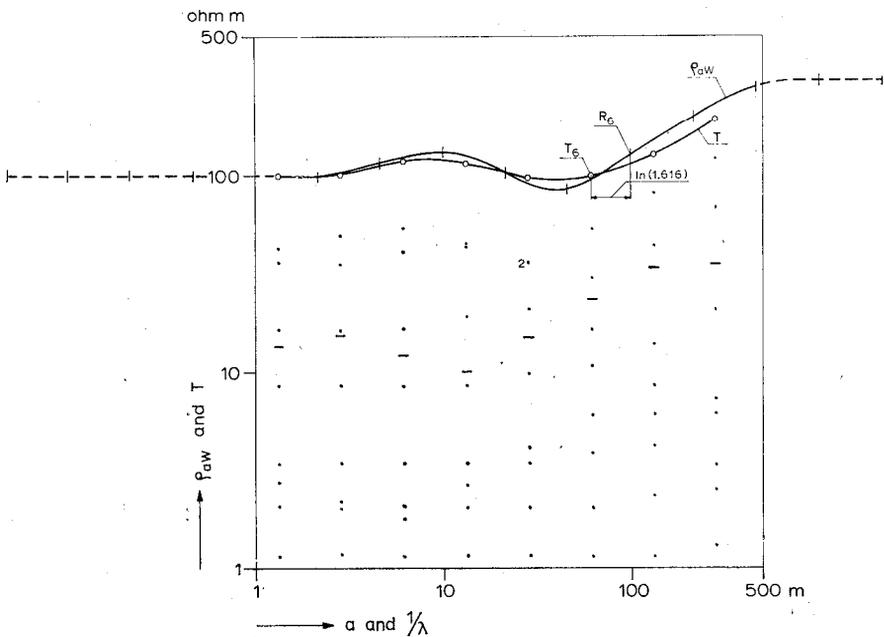


Fig. 8. Derivation of the transform curve from the sample values of a four layer Wenner apparent resistivity curve with $\rho_1 = 100$ ohm.m, $\rho_2 = 300$ ohm.m, $\rho_3 = 33.3$ ohm.m, $\rho_4 = 300$ ohm.m and $d_1 = 5$ m, $d_2 = 5$ m and $d_3 = 20$ m.

to the coefficients a_1 to a_6 and to the right to yield P, Q corresponding to a_{-1} and a_{-2} .

4. The operation of the coefficients on the inputs is performed by first superimposing the "resistivity" chart on to the "filter" chart with the point B on the cross and then tracing the filter points on to the resistivity chart. The procedure is repeated until all the other points (C to Q) have successively been at the cross. It is necessary to trace the filter points only in the range of the observed curve for which we desire to obtain the transform.

5. The traced points are then added up at each sample point and the dash is subtracted. The resultant value gives us the transform value which is then plotted.

6. Interpolation among the transform values gives us the T curve.

For the Wenner case only the first step of the above procedure need to be altered: it should now be

1. the filter coefficients given in table V are transferred to the filter chart in such manner that a_0 is plotted $\ln 1.616$ to the left of $y = 0$; the other coefficients are then plotted with respect to a_0 maintaining the constant spacing of $\frac{1}{3} \ln 10$. The cross is placed at $y = 0$ and ordinate 1. This is illustrated in fig. 7.

Steps 2 to 6 enumerated above for the Schlumberger arrangement are now executed for the Wenner arrangement.

As a consequence of altering step 1 the transform values will be automatically shifted to the left in relation to the resistivity values by $\ln 1.616$. This means that the transform curve will be extended to the left and shortened at the right by the above amount in relation to the resistivity curve (see fig. 8).

APPLICABILITY OF THE METHOD

The method is in general applicable to all forms of resistivity distributions within the limits of the theoretical assumptions on which Stefanescu's solution is based. As there is no loss of information in the transformation process, the question whether small layer differences will show up in the T curve depends to a large extent on whether such differences have been actually measured and are present in the resistivity curve.

An example is given here (fig. 9) to demonstrate the application of the method and any difficulty that may arise. In fig. 6 no such problem arises, as the asymptotic part of the curve is reached and the two extrapolated points to the right can easily be obtained.

However, in the example of fig. 9 the survey was abandoned when the curve was still descending rapidly. This is an example from a field survey performed in the Western part of the Netherlands by the Geoelectrical Workgroup of

TNO, a Government scientific research organization, to delineate the salt-freshwater boundary. In this particular problem there is no difficulty in extrapolation for the geology and working experience in the area furnish sufficient information as to the ultimate trend of the curve. This is one reason why unnecessary long spreads are avoided. Moreover, in problems such as this one errors in extrapolation can not have much influence on the output as the resistivity sample values of the extrapolated points are very low.

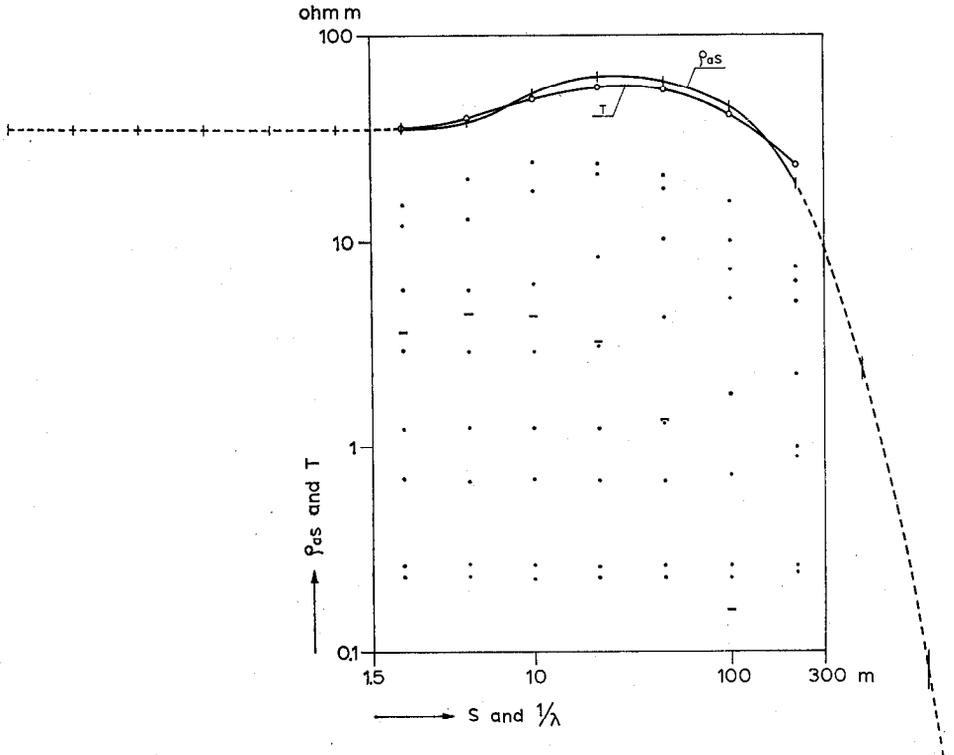


Fig. 9. Derivation of the transform curve from the sample values of a three layer Schlumberger resistivity curve obtained in the Western part of the Netherlands.

This can not be said for the example given in fig. 8 for the Wenner arrangement, because the trend of the curve is towards high resistivity and there could arise enough ambiguity in the manner the curve is to be extrapolated. For these types of cases the following suggestions are made:

1. to extend the survey,
2. or, alternatively, to use two layer standard curves asymptotic to the last part of the observed curve as a guide to careful extrapolation.

For the Wenner system either of the above suggestions could be adapted as a practice to compensate partly for the shortening effect of the derived transform curve. Lastly it is worth mentioning that the knowledge of the resistivity value of the substratum is also a requirement with other interpretational methods like, for instance, curve matching.

SOME FIELD CONSIDERATIONS

I. *Comparison of the spacing of the field data with the sampling interval*

The resistivity observations in the field correspond to increasing electrode spacings; the manner in which the electrodes are expanded in the field are different with different organizations, and also vary with the problem. It is even observed that in one particular survey not a constant ratio of expansion is used. One of the common electrode layout used in the Netherlands for the Schlumberger arrangement is as follows

$$s = 1, 1.5, 2.5, 4, 6, 8, 10, 15, 20, 25, 30, 40, 50, 60, 75, 100, 125, 150, 200, 250 \dots \text{ m}$$

ratio of expansion = 1.2 to 1.5.

Accepting an average ratio of expansion of 1.35 we shall see how this compares with the interval used for interpretation. For this purpose we need to convert the figures to the X -axis by the help of (8)

$$dx = \ln ds = \ln 1.35 = 0.3$$

For the Wenner arrangement it is customary to use a mode of expansion in which the potential electrodes occupy the positions vacated by the current electrodes in the preceding measurement such that we have

$$a = 1, 3, 9, 27, 81, 243, 729 \dots \text{ m}$$

ratio of expansion = 3

$$dx = \ln da = \ln 3 \approx 1.1$$

The sampling interval used for interpretation is

$$\Delta x = \frac{1}{3} \ln 10 \approx 0.77$$

From the above figures we see that the resistivity observations in the Schlumberger arrangement are much closer together than the interval suggested to sample them. On the other hand the spacing of the Wenner data is wider.

In principle on the basis of the frequency study performed in this paper it is clear that no extra information about the subsurface would be obtained by choosing intervals smaller than the suggested interval. Thus it might be pos-

sible to recommend a field procedure that should yield the field data exactly at an interval of $\frac{1}{3} \ln 10$ so that the data could be directly used for interpretation by the proposed method. However, due to an important consideration discussed below, it is not advisable to directly use the observations for interpretation.

2. *Inaccuracy of the field data*

It is well known that various factors influence resistivity observations in such a way that the data contain invariably contributions from sources other than what is to be expected from purely homogeneous and horizontal layering. The factors are

1. instrumental and observational errors termed as random noise,
2. errors caused by lateral surface inhomogeneties termed as geologic noise.

To get quantitative information of the noise content in a particular set of field data an energy density spectrum (see Lee 1960, Bracewell 1965) study of the data is necessary; this can be obtained from the Fourier spectrum. Qualitatively it can be said that with sensitive equipment and refined field techniques the first error could be kept to a minimum. The second effect is generally recognized by experience as a scattering of the observed values.

As interpretation of the resistivity data including the present method is based on ideal conditions, it is necessary to smooth out the data. Herein lies the justification of using close spacings, as in the Schlumberger field procedure. Moreover, it is worth mentioning that in the Schlumberger technique the observations are less sensitive to lateral inhomogeneties than with other sounding techniques like the dipole or the Wenner method. It can, however, be shown that with the type of filter response used here there is no possibility of noise amplification at the output.

3. *Utilization of all field information*

Our discussion in part 1 of this section showed that with the Schlumberger form of field procedure only about half of the field data is used in interpretation. Thus an alternative suggestion is given for those who like to utilize all their available information. Fig. 10 illustrates the procedure. The filter method is first applied to the resistivity sample values at the points marked with a dot and the transform obtained. The method is then repeated to a new set of sample values marked between the first set of values. In the figure they are defined by the circles and are themselves at a spacing of $\frac{1}{3} \ln 10$. The resultant transform values are thus obtained at a spacing of $\frac{1}{6} \ln 10$ i.e. 0.38, which compares very favourably with the field spacing of 0.3.

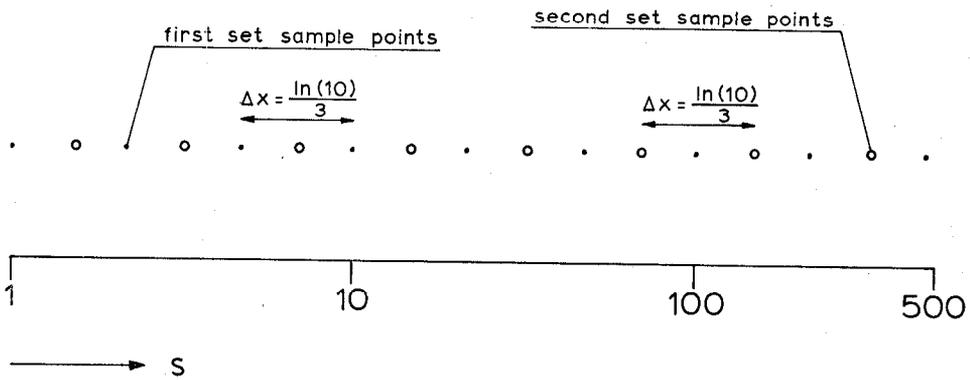


Fig. 10. Alternative procedure for utilization of all field information. The filter method is first applied to the resistivity values at the sample position described by the dots and then repeated to the second set of values at the sampling position denoted by the circles resulting in the derivation of the transform at a final spacing of $\frac{1}{6} \ln 10$.

ACCURACY OF THE METHOD

As there was no possible quantitative procedure of testing the accuracy of obtaining the transform values from the resistivity sample values, the method

ohm m.

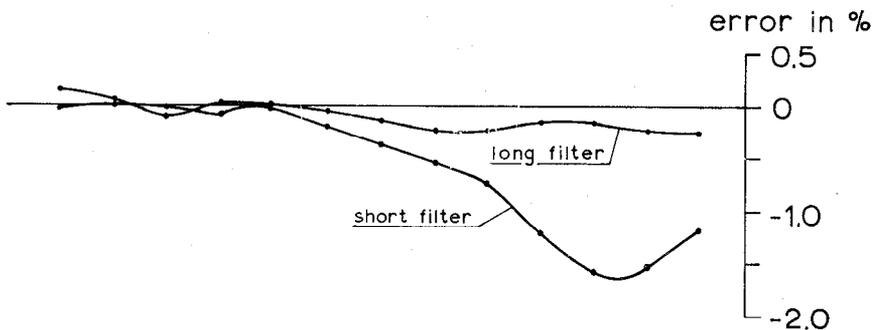
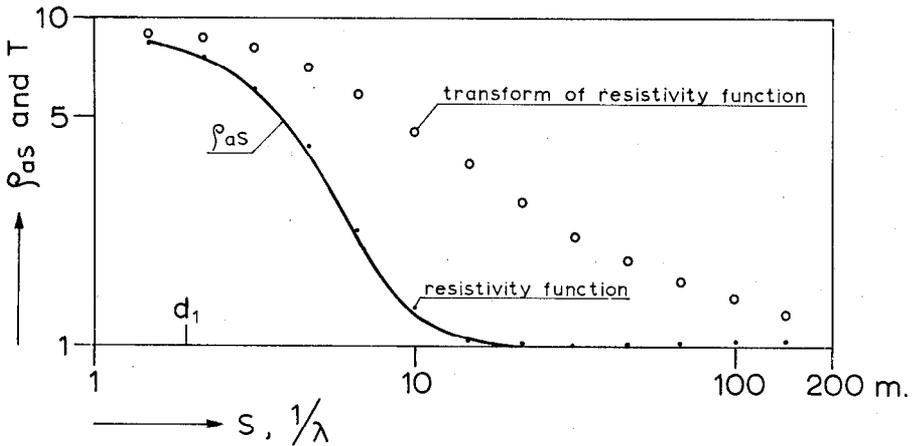


Fig. 11. Accuracy test. The circles in the upper part of the diagram are the transforms derived from the sample values of a partial resistivity function defined by the dots, at a spacing of $\frac{1}{6} \ln 10$. The full drawn resistivity curve shows the similarity of the function used with actual resistivity curves. The lower part shows the relative error at the sampling points between the derived and the theoretical transform.

was applied to partial resistivity functions which had close resemblance with actual field curves and whose theoretical transforms were known. It is thus to be expected that the cases discussed here, in connection with the accuracy tests, are more favourable than with field examples.

Various types of resistivity functions were chosen that had close similarity with ascending, descending and bowl-shaped maximum and minimum type resistivity curves. Their T values were determined by the filter method and the relative error calculated by comparing with their mathematical values. It was observed that with the bowl-shaped varieties the errors were very small (less than a percent), whereas with other types the errors were about 2%. These figures were obtained when the short filter was used. The long filter gave much smaller errors. Accuracies obtained with the short filter can be termed as quite satisfactory. Fig. 11 demonstrates one of the results of the tests.

SPEED OF APPLICATION

The graphical version of the method requires about a quarter of an hour to obtain the transform curve. The numerical calculation may take slightly longer. For those who feel this quarter of an hour is not a proper investment of time towards interpretation, the alternative procedure of interpretation or operation with the long filter can be tried out. These variations can easily be adapted by organizations that have at their disposal electric calculators with small memory.

In conjunction with Koefoed's (1970) method of deriving the layer distribution from the T curve, the whole sequence of physical interpretation can be completed in about half an hour. This should give a new meaning to the application of direct techniques in resistivity interpretation.

CONCLUDING REMARKS

1. The method suggested is simple in application; the graphical process is particularly suitable to the field geophysicist;
2. the method is applicable for both the Schlumberger and the Wenner form of field survey;
3. any form of logarithmic paper can be used if care is taken that the resistivity chart and the filter chart are in the same scale;
4. the speed and accuracy of the method are reasonably good;
5. the use of the transform function as the intermediary step in interpretation gives a clearer insight into the equivalence difficulties. The relation between ρ_a and T is a one to one relation, therefore no ambiguity can arise in this step. So in large scale surveys, the T curves may be derived and stored. As information from local geology or bore-holes become available, one can proceed with determining the layer parameters or alter them if they have already been evaluated.

ACKNOWLEDGEMENT

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APPENDIX I

Mathematical Expressions for the Partial Resistivity and their Corresponding Transform Functions

1. Relation between the Schlumberger and the Wenner resistivity functions

$$\Delta\rho_{aW} = 2a \int_a^{2a} (\Delta\rho_{aS}/s^2) ds$$

2. Functions used to determine the filter characteristic

First set:

$$\begin{aligned} \Delta\rho_{aS}(x) &= (\exp 3x)/(1 + \exp 2x)^{5/2} \\ \Delta\rho_{aW}(x) &= 2/3 [(\exp x)/(1 + \exp 2x)^{3/2} - (\exp x)/(1 + 4 \exp 2x)^{3/2}], \\ \Delta T(y) &= 1/3 e^{-y} \cdot e^{-e^{-y}} \end{aligned}$$

Second set:

$$\begin{aligned} \Delta\rho_{aS}(x) &= (\exp 3x)/(1 + \exp 2x)^{7/2}, \\ \Delta\rho_{aW}(x) &= 2/5 [(\exp x)/(1 + \exp 2x)^{5/2} - (\exp x)/(1 + 4 \exp 2x)^{5/2}], \\ \Delta T(y) &= (e^{-y} + e^{-2y})/(15 e^{e^{-y}}) \end{aligned}$$

3. Functions used to test the accuracy (fig. 111)

$$\begin{aligned} \Delta\rho_{aS}(x) &= 1 + 8(1 + \exp x + 1/2 \exp 2x)/(\exp(\exp x)), \\ \Delta T(y) &= 1 + 12/(1 + \exp 2y)^{1/2} - 4/(1 + \exp 2y)^{3/2}, \end{aligned}$$

APPENDIX II

Mathematical Expressions for the Resistivity Transform Function in Terms of the Layer Parameters

1. Two layer case

$$T(\lambda) = \rho_1 \cdot \frac{1 + k_1 e^{-2\lambda d_1}}{1 - k_1 e^{-2\lambda d_1}}$$

2. Three layer case

$$T(\lambda) = \rho_1 \cdot \frac{(1 + k_1 k_2 e^{-2\lambda d_2}) + (k_1 e^{-2\lambda d_1} + k_2 e^{-2\lambda(d_1 + d_2)})}{(1 + k_1 k_2 e^{-2\lambda d_2}) - (k_1 e^{-2\lambda d_1} + k_2 e^{-2\lambda(d_1 + d_2)})}$$

where ρ_i and d_i are the resistivities and thicknesses respectively of the layers concerned; the reflection coefficient k_i has its usual meaning.

APPENDIX III

Formula used to Reconstruct a Function from its Sample Values

$$g_r(x) = \sum_{m=-\infty}^{\infty} g_o(m \Delta x) \sin \frac{\pi(x - m \Delta x)}{\Delta x} \bigg/ \frac{\pi(x - m \Delta x)}{\Delta x}$$

where

$g_r(x)$ = reconstructed function,

$g_o(m \Delta x)$ = sample values of the original function $g_o(x)$,

Δx = sampling interval, and m = integer.

Relative error in reconstruction

$$\varepsilon = (g_o - g_r) / g_o$$

APPENDIX IV

Fourier Transform

Let $g(x)$ be an aperiodic function of the space variable x , then its representation in the frequency domain is given by the Fourier transform $G(f)$ as

$$G(f) = \int_{-\infty}^{\infty} g(x) e^{-i2\pi f x} dx \quad (17)$$

A sufficient but not necessary condition laid down for the existence of $G(f)$ is that the integral of $|g(x)|$ from minus to plus infinity is finite. $G(f)$ is in general a complex quantity and is described completely by its

amplitude density spectrum, $|G(f)| = (A^2(f) + B^2(f))^{1/2}$

phase density spectrum, $\varphi(f) = \arctan B(f)/A(f)$

where $A(f)$ and $B(f)$ are the real and imaginary components respectively of $G(f)$.

The Inverse Fourier transform converts the quantities back to its function domain

$$g(x) = \int_{-\infty}^{\infty} G(f) e^{i2\pi f x} df \quad (18)$$

(17) and (18) are different modes of representation of the same quantity and thus $g(x)$ and $G(f)$ are often referred to as a Fourier transform pair.

REFERENCES

- ANDERS, E. B., et al., 1964, Digital filters, NASA contractor report No. CR-136, Clearinghouse, Springfield, Virginia.
 BRACEWELL, R., 1965, The Fourier transform and its applications, McGraw Hill, New York.

- DEAN, W. C., 1958, Frequency analysis for gravity and magnetic interpretation, *Geophysics* 23, 97-128.
- DOBRIN, M. B. and PETERSON, R. A., 1966, A pictorial digital atlas, Presented at the 36th meeting of SEG at Houston, Texas, Nov. 1966; United Geophysical Corp. publication.
- KELLER, G. V., 1968, Electrical prospecting for oil, *Quarterly of the Colorado School of Mines* 63, No. 2.
- KOEFOED, O., 1968, The application of the Kernel function in interpreting geoelectrical measurements, *Geocexploration Monographs, Series 1, No. 2*, Gebrüder Borntraeger, Berlin-Stuttgart.
- , 1969, An analysis of equivalence in resistivity sounding, *Geophysical Prospecting* 17, 327-335.
- , 1970, A fast method for determining the layer distribution from the raised kernel function, *Geophysical Prospecting* 18, 564-570.
- LEE, Y. W., 1960, *Statistical theory of communication*, John Wiley, New York.
- PEKERIS, C. L., 1940, Direct method of interpretation in resistivity prospecting, *Geophysics* 5, 31-42.
- PICKLES, E., 1967, *Lecture notes on the basic mathematics of digital processing of seismic data*, G.S.I., Inc. (USA) publication.
- ROBINSON, E. A. and TREITEL, S., 1964, The stability of digital filters, *IEEE Transactions on Geoscience Electronics, Vol. GE-2*, 6-18.
- ROBINSON, E. A., 1967, *Statistical Communication and Detection*, Charles Griffin and Co. Ltd., London.
- SLICHTER, L. B., 1933, The interpretation of the resistivity prospecting method for horizontal structures, *Physics* 4, 307-322.
- STEFANESCO, S. S., et al., 1930, Sur la distribution électrique potentielle autour d'une prise de terre ponctuelle dans un terrain à couches horizontales homogènes et isotropes, *Journal de Physique et du Radium, Series 7*, 132-140.
- SWARTZ, C. A. and SOKOLOFF, V. M., 1954, Filtering associated with selective sampling of geophysical data, *Geophysics* 19, 402-419.
- WATSON, G. N., 1966, *A treatise on the theory of Bessel functions*, Cambridge University press, 2nd. edition reprinted.
- WEBER, M., 1964, Ein direktes Verfahren zur Interpretation von geoelektrischen Messungen nach Schlumberger, *Pure and Applied Geophysics* 59, 123-127.