The theoretical basis for prestack migration by equivalent offset
Gary F. Mar-grave* and John C. Bancroft, The CREWES Project, The University of Calgary

Summary

The method of prestack migration by equivalent offset (EOM) is reviewed and compared to post stack migration with DMO processing. The method forms common scatter point (CSP) gathers for each migrated trace and then images those gathers with a migration algorithm. The Fourier dual algorithm to EOM, called equivalent wavenumber migration or EWM, is derived from Fourier migration theory. EWM is found to be an exact reformulation of prestack FK migration. The CSP gathers are shown to be formed by a Fourier mapping and inverse transform of the unmigrated spectrum. Then each CSP gather is imaged by an algorithm which is formally identical to post stack migration with the result retained only at zero equivalent offset. The EWM and EOM algorithms are then argued to be the Fourier and Kirchhoff duals of one another.

Introduction

The modern theory of seismic wavefield imaging (migration) is generally acknowledged to rest on theoretical developments from the late 1970’s and early 1980’s such as Stolt (1978) Schneider (1978) and Gazdag (1978). Conventional seismic data processing is usually separated into prestack and poststack processes where “stack” refers to the common midpoint (CMP) stacking technique. Though seismic data is manifest a wavefield, wavefield imaging techniques are usually confined to the poststack realm for economic and other practical reasons in spite of the general recognition that prestack migration is theoretically preferable. This has lead to the development of DMO (dip moveout) theory which improves the conventional image by improving the input to poststack migration. Hale (1983) put DMO theory on a firm theoretical basis by deriving its relationship to the prestack migration theory formulated in Stolt (1978). Hale proved that, for constant velocity, prestack migration is fully achieved by the cascade of three imaging steps: NMO removal, DMO correction, (stack) and poststack migration. Extension of the DMO theory to non-constant velocity has proven possible for v(z) (i.e. “time migration”) but problematic for v(x,z). Nevertheless, the theory has been of great practical benefit to seismic exploration.

Bancroft and Geiger (1994) and Bancroft et al. (1995) introduced an alternative technique initially called common scatterpoint (CSP) migration and now called equivalent offset migration (EOM). The essence of this technique is to bypass CMP stacking completely by forming a new kind of gather which assumes a common subsurface scatter point rather than a common source-receiver midpoint. The traveltime expression from source to receiver via a scatterpoint at x=0 (in a constant velocity medium) is called the double square root (DSR) equation and is written in terms of midpoint, x, and half-offset, h, in eqn (1). This equation also defines “equivalent offset”, h_e, by asserting that the DSR can be written as a single square root. Bancroft et. al showed that he is given exactly by (2). It is well known that the traveltime surface described by (1) is not a hyperboloid in (x,h) but instead has a quasi-rectangular cross section and is commonly called Cheop’s pyramid (Claerbout 1985). The equivalent offset reformulation amounts to a coordinate transformation which maps Cheop’s pyramid into a hyperbola in he (for each x) which is then easily imaged with conventional (i.e. poststack) migration theory.

\[ t = \sqrt{\left(\frac{1}{2}t^2_v\right) + (x+h)^2} + \sqrt{\left(\frac{1}{2}t^2_v\right) + (x-h)^2} = 2\sqrt{\left(\frac{1}{2}t^2_v\right) + h_e^2} \]  

(1)

\[ h_e^2 = x^2 + h^2 - \frac{4x^2h^2}{v^2} \]  

(2)

By assuming that a vertical alignment of common scatterpoints will result in a set of traveltime surfaces with the same vertical alignment, the theory was restricted to “time migration” but a practical imaging algorithm emerged. Prestack migration by equivalent offset is done by first forming CSP gathers for each desired migrated trace (scatterpoint position) and then imaging those gathers. Each input trace contributes to all CSP gathers within the migration aperture and is mapped at constant time to a spatial position in each gather given by the trace’s equivalent offset. The velocity dependence of he is not strong and is easily handled by the common iterative approach of assuming a trial velocity function and later refining that guess. As data is mapped into the CSP gathers, it is binned at some sensible bin size (usually half of the receiver group interval) and thus a great reduction in data volume occurs. Since the traces are mapped at constant time, no expensive trace interpolations are required (though static corrections should already be applied). Equivalent offsets are never less than source-receiver offsets (and often are much greater) so CSP gathers are much more sensitive velocity analysis instruments than CMP gathers. Once final
Theory of equivalent offset migration

velocities are determined (and the gathers reformed if the initial guess velocity was wildly wrong) the final imaging is done by migrating each gather with an algorithm identical to post-stack migration and evaluating (retaining) the migrations only at zero equivalent offset. Thus another large saving over conventional prestack Kirchhoff techniques is realized because the time consuming steps of dip dependent scaling and antialias filtering are performed on the CSP gathers. The first order approximation to this imaging step is conventional NMO correction and stacking of the CSP gathers.

Next, we present a formal justification of EOM by showing that, in the constant velocity case, it is equivalent to prestack migration as formulated by Stolt (1978) in the Fourier domain. This is done by deriving, from Stolt’s equations, the Fourier parallel to EOM, which we call equivalent wavenumber migration (EWM), and then showing that EOM is a conventional Kirchhoff approximation to EWM while the latter is exactly equivalent to prestack migration in the Fourier domain. Thus the theoretical basis for EOM is at least as good as that of NMO-DMO-poststack migration. The extension of both algorithms to the practical case of non-constant velocity is approximate though we believe there are a number of practical advantages to EOM (as mentioned above).

Sketch of the Equivalent Wavenumber Algorithm

Let \( \Psi_0(x,h,t) \) represent the prestack data for a 2-D experiment in midpoint and half-offset coordinates. Then its 3-D Fourier transform is:

\[
\hat{\Psi}_0(k_x,k_y,\omega) = \int \int \int \Psi_0(x,h,t) \exp\{-i \omega t - ik_x x - ik_y h\} dk_x dk_y d\omega
\]  

(Note that we neglect the Fourier transform constant scale factors). Stolt’s prestack theory then leads to the following expression for the “Stolt wavefield”:

\[
\Psi(x,h,t,z) = \int \int \int \hat{\Psi}_0(k_x,k_y,\omega) \exp\{i \omega t - ik_x x - ik_y h\} dk_x dk_y d\omega
\]

where:

\[
k_z = \sqrt{\frac{2\omega}{v}} \sqrt{(k_x - k_h)^2 + \frac{1}{2} \sqrt{\frac{2\omega}{v}} (k_x + k_h)^2 - (k_x + k_h)^2}
\]

The Stolt wavefield is a four dimensional construct that can be evaluated to yield either the prestack data or the migrated section as follows:

\[
Y(x,h,t,z=0) = \Psi_0(x,h,t) = \text{the prestack data; and } Y(x,h=0,t=0,z) = \text{the prestack migrated section}
\]

These expressions are presented in essentially the notation of Hale (1983). The vertical wavenumber, \( k_Z \), as given by (5), is found from a double square root equation which is the Fourier dual of equation (1). In precisely the same way as the time domain derivation (Bancroft and Geiger 1994, Bancroft et al. 1995), the Fourier DSR can be rewritten as a single square root involving a new “equivalent” wavenumber. Thus we define equivalent wavenumber, \( k_e \), implicitly through:

\[
\frac{1}{2} \sqrt{\frac{2\omega}{v}} (k_x - k_h)^2 + \frac{1}{2} \sqrt{\frac{2\omega}{v}} (k_x + k_h)^2 = \frac{2\omega}{v} - k_e^2
\]

Ordinary, though tedious, algebra leads to:

\[
k_e^2 = \frac{1}{2} (k_x^2 + k_h^2) - \frac{1}{2} \sqrt{(k_x^2 - k_h^2)^2 - 4k_x^2 k_h^2} = k_x^2 + k_h^2, \text{ where } k = \frac{2\omega}{v}
\]

Note that (8) presents two equivalent forms for \( k_e \), the first being better suited for our derivation but the second is more comparable to (2). We proceed by changing the Fourier integration variables in (4) from \((k_x,k_h,k)\) to \((k_x,k_e,k)\).

The results of the lengthy derivation are:

\[
\Psi(x,h=0,t=0,z) = \frac{v}{2} \int \int \phi(0,x,k_x,\omega) \exp\{i \sqrt{k^2 - k_e^2} z\} dk_x dk
\]

where:

\[
\phi(k_x,\omega) = \frac{1}{2} \int f(k_x,k_x,k) \hat{\Psi}_0(k_x,k_x,k) \exp\{i k_x x\} dk_x
\]

and:

\[
\hat{\Psi}_0(k_x,k_x,k) = \hat{\Psi}_0(k_x,\omega) \exp\{i k_x x\}
\]

\[
\hat{\Psi}_0(k_x,\omega) = \hat{\Psi}_0(k_x,\omega) \exp\{i k_x x\}
\]
Theory of equivalent offset migration

and:

\[ f(x_h,k_h,k) = \frac{k_x}{k_h} \left( 1 - \frac{k_x^2 - k_h^2}{k_x^2 + k_h^2 - k_e^2} \right) \]

\[ \text{with } k_h = \text{sign}(k_e) \sqrt{\frac{(k_x^2 - k_h^2)(k_e^2 - k_h^2)}{k_x^2 - k_e^2 + k_h^2}} \]  

Equation (9) is an exact reformulation of (4) and thus represents a solution to the constant velocity migration problem of the same accuracy as Stolt’s FK theory. The function \( \Phi(x,k_e,k) \) as given by (10) is the Fourier transform (2D) of CSP gathers at location \( x \). Equations (11) and (12) show that these gathers are computed by mapping at constant temporal frequency (i.e. \( k \)) the spectrum of the unmigrated data, \( \Phi(k_x,k_h,k) \) from \((k_x,k_h)\) space to \((k_x,k_e)\) space and applying a scaling operation, followed by an inverse Fourier transform over \( k_x \). Though algebraically complex, the mapping operation is conceptually simple when depicted graphically as in figure 1. Note that we map \( kh \) to \( ke \) of the same sign. The scaling operation is simply that required to conserve total spectral power under the mapping.

Equation (9) says that the CSP gathers are imaged to form the migrated section at constant \( x \) by a process resembling NMO and stack. In fact, we can consider (9) to be the limiting case of a Stolt-like wavefield in which the equivalent offset, rather than the half-offset, is set to zero. That is, define the “Stolt equivalent” wavefield:

\[ \Psi(x,h,t,z) = \frac{1}{2} \int \Phi(x,k_e,k) \exp \left( i \sqrt{k_x^2 - k_e^2} z \right) \exp \left( -i k_v t / 2 + i k_h k_e \right) dk_e dk \]  

and note that equation (9) results from:

\[ \Psi(x_h=0,t=0,z) = \Psi(x,h=z=0,t=0,z) \]  

The formal similarity between (13) and the post-stack migration equations is striking. In fact, setting \( t=0 \) in (13) gives an identical expression to post-stack Stolt migration for each \( x \). Thus (13) does indeed say to move energy along hyperbolae in equivalent offset.

A stationary phase analysis of (13) will be shown to demonstrate that he defined here is the same as that defined in the time domain by Bancroft et al. We will also show (with a numerical simulation) that the impulse response of the CSP gathers formed in EOM theory and the gathers given by (10) are the Kirchhoff and Fourier duals of one another. Figure 2 summarizes these results.

Conclusions

Prestack time migration may be performed, with high precision and efficiency, by the process of sorting the data into CSP gathers at equivalent offset and then imaging those gathers. This process, called EOM, is the \((x,t)\) domain Kirchhoff approximation to a Fourier theory, called EWM, which is an exact reformulation of Stolt’s (1978) constant velocity algorithm. Both EOM and EWM are based on the recognition that the double square root equations underlying each can be written exactly as single square roots which define equivalent offset and equivalent wavenumber. The EWM process forms ‘the CSP gathers in the Fourier domain by mapping and scaling the 3D spectrum of the 2D prestack wavefield. Imaging of the resultant gathers may be done exactly by a post stack migration of each and retaining only zero equivalent offset or approximately by NMO removal and stacking.

Acknowledgments

We wish to thank the sponsors of the CREWES project for their support of this research.

References

Claerbout, 1985, Imaging the Earth’s Interior, Blackwell Scientific Publications
Gazdag, J., 1978, Wave-equation migration by phase shift, Geophysics, 43, 1342-1351
Schneider, W., 1978, Integral formulation for migration in two and three dimensions, Geophysics, 43, 49-76
Fig. 1. On the left is a map of \((k_x, k_h)\) space for (for constant \(k\)) and on the right is \((k_x, k_e)\) space created by the mapping of equation (11). Essentially, each contour on the right splits in two, along the \(k_h=0\) axis, with the upper part becoming a horizontal line in the upper triangular region on the left and similarly for the lower part. Shading and contours in both maps indicate the value of \(k_e\) though the shading levels are different as noted.

Fig. 2. On the left is a graphical representation of the impulse response of EOM. First a single sample at \(x=0, h=500, t=2.0\) is mapped into \(h_e\). This forms the hyperbola in the \(t=2.0\) plane which is the impulse response of CSP gather formation (found by setting \(h = \text{constant}\) in eqn (2)). Then each point on the horizontal hyperbola is replaced with a wavefront circle representing the imaging of each CSP gather. The final impulse response is found in the \(h_e=0\) plane and is the well known ellipse from prestack migration theory. On the right is a plan view of the \(t=2.0\) second plane showing the formation of CSP gathers with the Fourier method of equation (10). Shown as a dashed line is the ray theoretic response from the left figure. It has been shifted up for convenience of comparison and actually overlays the Fourier image precisely.