Converted wave migration and common conversion point binning by equivalent offset
Xinxiang Li* and John C. Bancroft, The CREWES Project, The University of Calgary

SUMMARY
A method for converted wave prestack time migration is presented. This method splits the conventional prestack Kirchhoff time migration into two steps using equivalent offset binning. Migration gathers are first constructed instead of producing the migrated section directly. This step also completes the common reflection point binning. Conventional NM0 and stacking applied on these gathers complete the imaging process.

The main difficulty in implementing this method is the accurate computation of the equivalent offset. An approximate algorithm and its accuracy analysis are presented in this abstract.

INTRODUCTION
The concept of equivalent offset was first introduced to perform prestack Kirchhoff time migration by two separate steps. The first step is the construction of a set of migration gathers sorted by the common scatter point (CSP) surface locations and the equivalent offsets. The second step consists of conventional velocity analysis, normal moveout (NMO) correction and CDP stacking. The CSP gathers provide higher-fold and higher signal to noise ratio data to obtain dip-independent migration velocity. The applications of this method have obtained very good results on velocity analysis and final imaging. (Bancroft and Geiger, 1994)

For converted waves, the CMP gathers are not the common conversion point (CCP) gathers even for layered subsurface structure. The CCP binning is a very initial step in regular converted wave seismic data processing (Tessmer and Behle, 1988). Equivalent offset migration (EOM) is based on scatter point model, the reflection point dispersal due to the wave mode conversion can also be attenuated by the constructive and destructive behavior of the equivalent offset binning, as it does for the P-P wave. Some basic work has been done by Wang, Bancroft and Lawton (1996).

KINEMATICS OF CONVERTED WAVES
The two way traveltime for a scatter point at \( (x, z) \) can be described as

\[
\tau = \frac{1}{v} \sqrt{(x+h)^2 + z^2} + \frac{1}{\gamma v} \sqrt{(x-h)^2 + z^2},
\]

where \( h \) is the half source-receiver offset, \( v, \gamma v \) are the velocities (Figure 1). For simplicity, we suppose that the source-receiver midpoint is at \( x = 0 \) and \( z = 0 \).

THEORY
At a scatter point, we can define compressional and shear wave averaged velocity as

\[
v_a = \frac{2v}{1+\gamma} v,
\]

It satisfies \( z = v_a \tau / 2 \), where \( \tau \) is the zero offset time at the surface location above the scatter point. Using \( v_a, \tau \) and

\[
E = (v)(1 - \frac{4\beta ch}{(v^2)^2})
\]

equation (2) can be expressed as
EOM and binning for converted waves

\[(v_x \tau)^2 = (v_x t)^2 - 4 \left[ \frac{1}{\beta} (x + h)^2 - \frac{1}{\beta y^2} (x - h)^2 - \frac{2E}{\beta^2 y^2} \right]. \quad (5)\]

For the sample with travel time \( t \) on the trace with source-receiver half offset \( h \) and CMP location \( x = 0 \), we can define the equivalent offset \( h_e \) for the CSP location \( x \) as

\[h_e^2 = \frac{1}{\beta} (x + h)^2 - \frac{1}{\beta y^2} (x - h)^2 - \frac{2E}{\beta^2 y^2}. \quad (6)\]

This makes equation (5) hyperbolic:

\[t^2 = \tau^2 + \frac{4h_e^2}{v_a^2}. \quad (7)\]

Combine equation (1) and (7) together, we have

\[2 \sqrt{\frac{h_e^2 + z^2}{v_a}} = \frac{v}{\sqrt{\frac{(x + h)^2 + z^2}{v_a}} + \frac{(x - h)^2 + z^2}{\gamma v}}. \quad (8)\]

Now the equivalent offset can be explained as the distance between a pair of co-located source-receiver and the CSP surface location as shown in Figure 2. This shows that our definition is identical to that of Wang, Bancroft and Lawton (1996).

Figure 2. The definition of equivalent offset for a sample. The travel time from the source \( S \) to the scatter (with velocity \( v \)) then to the receiver \( R \) (with velocity \( \gamma v \)) is equal to the two way travel time from the co-located source-receiver \( S'R' \) to the scatter point (with velocity \( v_0 \)).

Figure 3 shows how the equivalent offset migration (EOM) works for converted waves for a constant velocity model. For comparison, (a) shows the converted wave Kirchhoff migration response of a sample on an input trace. The stretched “ellipse” is the image on the migrated time section. (b) and (c) show how the sample energy is distributed during the construction of CSP gathers, in \((x,t)\) and \((x,h)\) domain respectively. Instead of distributing the energy directly to the stretched ellipse on migrated section with different amount of time shift at different CSP locations, EOM distributes the energy without any time shift, but changes the offset to different equivalent offsets at different CSP locations instead. Besides the asymmetric property due to the wave mode-conversion, it is important to mention that the minimum equivalent offset is usually less than the source-receiver offset \( h \), which is always equal to \( h \) in P-P wave case.

The definition of equivalent offset is sample-based, but in practical computation, it is unnecessary to compute the offset for each sample. In fact, a series of offset bins and their boundaries are given at each CSP location, the problem left is to find the corresponding samples for each of these offset bin boundary values. The physical continuity ensures that the samples between each adjacent pair of these boundary samples have equivalent offsets falling in the same offset bin.

From the definition of equivalent offset (6), it is impossible to obtain an expression for the travel time \( t \) because \( \tau \) (zero offset two way travel time) at which \( v \) and \( \gamma \) are given also depends on the unknown travel time \( t \). An approximate numerical solution for computing equivalent offset is presented in the following two sections.

**APPROXIMATION AND ACCURACY ANALYSIS**

The approximate solution is based on the polynomial expansion of the square root term in equation (4). By second order Muir expansion (Claerbout, 1985),

\[E = 2\beta x h + 2\beta^2 \chi^2 h^2 + \ldots \left( \frac{v}{\gamma} \right)^2 - \beta x h \]

Introducing a parameter \( \theta = 1 - \beta x h / (\gamma v)^2 \), equation (6) becomes
The fit order approximation (the first three terms) is also very useful for practical computation. The justifier \( \theta \) can be used for simplifying the computations and maintaining the accuracy.

The velocity \( v \), the velocity ratio \( \gamma \) and the justifier \( \theta \) are the quantities that we can not always obtain their accurate values. Their effects to the accuracy of the equivalent offset computation are discussed.

The analysis of EOM method on P-P data by Bancroft and Geiger (1996) tells that the construction of CSP gathers is very insensitive to the velocity field. A similar conclusion applies to converted wave case, the accuracy of equivalent offset computation is also insensitive to the velocity, especially at deeper part of a trace or when the velocity is high. An example result is shown in Figure 4(a), where, even the velocity changes from 2000m/s to 4000m/s, the difference of equivalent offsets (at time range 1.0s to 2.0s) is still less than 5m. However, as part of the velocity information, the velocity ratio has significant influence on the equivalent offset. As in Figure 4(b), where the velocity ratio changes from 1.5 to 3.0, the range of equivalent offsets (at time range 1.0s to 2.0s) is from 310m to near 350m.

From equation (2) we have a built-in condition about the range of the CSP to CMP distance \( x \), that is

\[
4gh < \left(\frac{v}{\gamma}\right)^2.
\]  

The physical meaning of this condition is, for a given sample on an input trace with travel time \( t \), no scatter point, with the distance from its surface location to the CMP location further than the range of \( x \) defined by (10), has energy contributed to this sample. Equation (10) also gives a condition on the justifier \( \theta \), i.e.,

\[
\frac{3}{4} < \theta < 1.
\]  

The effect of the justifier to the accuracy of equivalent offset computation is also examined by the example model used for Figure 4. The result is shown in figure 5.

In general, at later time (such as later than 1.0S), the \( \theta \) should equal to 1.0, and smaller at early times. The \( \theta \) mainly depends on the relative ranges of the offset \( h \) and the CSP-CMP distance \( x \) to the velocity.
EOM and binning for converted waves

Practical experiments suggest a choice of a time variant $\theta$ as a piece-wise straight line:

$$\theta(t) = \begin{cases} 
0.75 + 0.3\tau, & \text{for } 0 \leq \tau \leq 0.5, \\
0.9 + 0.2(\tau - 0.5), & \text{for } 0.5 \leq \tau \leq 1.0, \\
1.0, & \text{for all } t \geq 1.0.
\end{cases}$$

For relatively small $h$ and $\tau$, it can be bigger at the first two parts, while for big $h$ and $\tau$, it can be smaller.

PRACTICAL COMPUTATION

The computation starts with a given equivalent offset bin boundary, also denoted by $h_e$, the purpose is to compute a value of the travel time $f$ from equation (9) as a sample time boundary. Our solution of this problem can be expressed as following:

(i) Obtain a preliminary velocity ratio from the first order approximation of equation (9),

$$\gamma_a = \frac{h_e^2 - (x-h)^2}{(x+h)^2 - h_e^2}. \quad (12)$$

(ii) Find the largest $\gamma$ smaller than $\gamma_a$ from the given velocity ratio function.

(iii) Combine (9) with (7) to cancel the term $\{vt\}$, we have

$$\theta(v_a, \tau) = \frac{16\tau^2 h_e^2}{x^2 + h_e^2 + 2xh - h_e^2 \left(1 + \gamma\right)^2 - 4h_e^2}. \quad (13)$$

All variables at the right hand side of this equation are known, and accurate except $\gamma$. $\theta(v_a, \tau)$ an absolutely increasing function of $\tau$ for RMS type velocity $v_a$ (Li and Bancroft, 1996). So, (13) can give a $\tau$ and then the velocity $v_a$ at this $\tau$.

(iv) Use the $\tau$ and $v_a$ obtained above, by (7) we compute the travel time $f$.

Notice that $\gamma$ is also a function of $\tau$, usually not constant, from step (iii), the $\gamma$ at the estimated $\tau$ might be used for equation (13) to obtain a new $\tau$ and $v_a$.

This iterative technique can be useful for early travel time estimation.

CONCLUSION

The basis of EOM for converted wave is stated in detail. We addressed how EOM completes conventional Kirchhoff migration in a different way, and how the construction of CSP gathers automatically completes the CRP binning. The main part of the work is the definition and computation of equivalent offsets. The accurate formula yields to the approximate solution because of the asymmetric kinematics. Simple synthetic experiments prove that our approximate solutions are reliable.

ACKNOWLEDGMENT

The authors are grateful for the financial support provided by the CREWES sponsors.

REFERENCES


