A theoretical comparison of equivalent-offset migration and dip moveout–prestack imaging

J. Bee Bednar*

ABSTRACT

Equivalent-offset migration is a methodology for prestack-Kirchhoff time migration that partially reverses the order of velocity analysis, normal moveout correction, stack, and migration. Though claimed to be computationally and analytically superior to earlier time-domain approaches, it is not independent of velocity. Velocity-independent dip moveout followed by prestack imaging is similar to equivalent-offset migration in that it also postpones normal moveout correction to the postmigration stage, but it is independent of velocity. This paper investigates the theoretical relationship between these two processes, showing that equivalent-offset migration and prestack imaging are asymptotically equivalent. Moreover, equivalent-offset migration has little, if any, computational or analytic advantage. The fact that the combination of dip moveout followed by prestack imaging is independent of velocity is a major advantage and suggests that this latter method is better suited to problems of velocity estimation.

INTRODUCTION

The purpose of seismic migration is accurate focusing of surface measurements at subsurface reflection points. Straight-ray approximations degrade the ability of time-migration techniques to produce accurate subsurface images. Relaxing constant velocity constraints by allowing velocities to vary as functions of midpoint and time does not resolve the issue. Rays are still forbidden to bend and, as a result, data are not properly focused.

Modern depth-imaging techniques allow rays to bend and are understood well enough to have become a standard part of data processing. They are rapidly leading to sophisticated methods for velocity estimation and mapping. One might conclude that further work on time-based algorithms is unnecessary, but depth migrations do have an Achilles heel. They require an acceptable initial velocity model prior to application. Without good initial velocity estimates, iterative depth migration can lead to disastrous results. Although estimation of prior velocity information is the subject of considerable discussion and research, approaches can be formulated to postpone velocity estimation until the data have been fully migrated.

The basic idea is to reverse the conventional processing sequence (1) velocity analysis, (2) normal moveout (NMO) correction, (3) dip moveout (DMO), (4) stack, and (5) poststack migration, to something like (1) prestack migration, (2) velocity analysis, (3) NMO correction, and (4) stack.

In the latter approach, velocities are chosen after migration and so, in principle, they are independent of dip. They are also estimated along image rays and, therefore, pertain more to true migrated positions than methods that do not incorporate migration in their formulation. A stacking-velocity field derived from migrated data provides an improved model for initial depth-imaging requirements.

There are several candidate techniques for reversing the processing sequence. An excellent review of the alternatives is presented by Fowler (1997). Prestack time migration of NMO-corrected common-offset sections followed by inverse NMO can be used to perform residual velocity analysis and, in a sense, provides a crude approach to the second processing sequence described above. Gardner et al. (1986) develop a two-step dip moveout (DMO) and prestack imaging (PSI) approach for forming common scatterpoint gathers. They claim that their method is completely velocity independent, and fully reverses the traditional processing sequence. Berryhill’s (1996) nonimaging–shot-record migration is similar to Gardner et al.’s two-step approach; in fact, Popovici (1994) claims that the two methods are equivalent. Moreover, Popovici (1994) shows that Berryhill’s method is computationally much faster. This is an important issue for subsequent remarks and should be kept in mind as the paper progresses. The method described by Ferber (1994) and Ferber et al. (1996) provides an additional approach to reversing the processing sequence, but is also
similar in spirit to that of Gardner et al. Since it concentrates more on 3-D issues, it is of significant interest for that alone. Bancroft and Geiger (1994) and Bancroft et al. (1995) introduced equivalent-offset migration (EOM), suggesting that it is several orders of magnitude faster than more traditional Kirchhoff approaches. They (Bancroft and Geiger, 1994; Bancroft et al., 1995; Bancroft, 1997) claim that a major benefit of their method is the production of high-fold, large-offset common scatterpoint gathers that improve the focusing of velocity semblance, resulting in improved velocity estimates. They suggest that rough initial velocity estimates are the only requirement for achieving this result. Bancroft (1997) further claims that the relationship between EOM and DMO followed by PSI (DMO-PSI) is superficial.

In view of the similarities, this paper compares the Gardner et al. (1986) method with Bancroft and Geiger (1994), Bancroft et al. (1995) and Bancroft (1997) by first casting each within the same framework and notation. For the reader’s convenience, velocity-independent dip moveout, prestack imaging, and equivalent-offset migration are each explained in separate sections using the same theoretical foundation. It is then clear that equivalent-offset migration only partially reverses the standard processing steps. The EOM schematic is velocity analysis, prestack migration, NMO, and stack. Moreover, EOM is asymptotically equivalent to the PSI step in DMO-PSI. On the other hand, dip moveout followed by prestack imaging is a full reversal of the standard processing sequence. It is completely velocity independent. Both PSI and EOM perform the imaging step on constant time slices by diffracting energy over appropriately defined curves. Not surprisingly, DMO can be implemented in exactly the same manner. The PSI curves are much simpler than those for EOM, and so are significantly easier to implement. Moreover, DMO-PSI can be formulated as a Fourier domain process, and so has computational complexity of the form \( n \log n \), where \( n \) is either the number of midpoints, the number of offsets, or the number of samples after a suitable log stretch. Basically, one is required to perform a double fast Fourier transform over midpoint and log stretch time as well as a Fourier transform over offset. Usually, the transform computations are dominated by the number of samples in the log stretch.

Although all the results in this paper focus on 2D relationships, extension to 3D is straightforward.

**VELOCITY INDEPENDENT DMO EQUATIONS**

In the early 1980s, G. H. F. Gardner and colleagues at the University of Houston showed that it was possible to DMO-correct recorded data in a velocity-independent manner prior to NMO or migration. The fundamental basis for Gardner’s method for velocity-independent DMO is the recognition that the double-square-root equation can be converted into a single-square-root equation by a radial-plane-diffraction process. After application of DMO, a second diffraction process, which takes place on constant time slices, completes the prestack migration step. These results were finally published in Gardner et al. (1986).

Consider a constant velocity medium with velocity \( v \). With reference to Figure 1, the double-square root equation is:

\[
T = \sqrt{T^2_M + \frac{(x + h)^2}{v^2}} + \sqrt{T^2_M + \frac{(x - h)^2}{v^2}}.
\]

Here \( x = m - m_0 \), \( m \) is the midpoint between source \( S \) and receiver \( R \), \( m_0 \) is the arbitrary location of a fixed scatter point, \( h \) is the half-offset, \( T_M \) is the vertical or migrated traveltime, and \( T \) is the time required to traverse the path SPR. The ellipse in Figure 1 is the locus of all points \((x, y)\) for which \( T \) is constant. Note that this is the case, both \( m_0 \) and \( m \) are referenced to the same fixed origin while \( x \) is relative to the midpoint \( m \). Migration collapses the amplitudes along the double square root into a single point directly below \( m_0 \) at time \( T_M \). Accomplishing this without knowledge of the velocity is performed in three steps. The double square root is collapsed into a single-square-root equation, the single square root is collapsed into a single hyperbolic curve at \( m_0 \), and the hyperbolic curve is NMO-corrected and stacked. The starting point is an equation which expresses \( T \) in equation (1) in terms of the normal incidence time from \((b, 0)\) in Figure 1 to the point \((x, y)\).

Following Forel and Gardner (1988), a brief algebraic manipulation yields

\[
\left(1 - \frac{b^2}{h^2}\right)T^2 = T_0^2 + \frac{4h^2}{v^2}\left(1 - \frac{b^2}{h^2}\right). \tag{2}
\]

where \( T_0 \) is the one-way traveltine from the point \((b, 0)\) in Figure 1 along the normal to the point \((x, y)\) on the ellipse.

Now define a new traveltine \( T_1 \) and offset \( h_y \) as follows:

\[
T_1 = \left(1 - \frac{b^2}{h^2}\right)T^2 \tag{3}
\]

and

\[
h_y^2 = h^2 - b^2. \tag{4}
\]

Then after rearranging equation (2), one has

\[
T_1 = \frac{h^2}{h_y}T = T_0^2 + \frac{4h^2}{v^2}. \tag{5}
\]

Equations (3), (4), and (5) define velocity-independent DMO. Equation (4) and the first half of equation (5) provide the framework for mapping traces with offset \( h \) and midpoint \( m \) to zero-offset traces with offset \( h_y \) and midpoint \( m + b \). Equation (3) provides a method for mapping from time \( T \) to time \( T_1 \) in a fixed constant offset plane. In either case, the result is a new data set with hyperbolic moveout based on a single-square-root.

**FIG. 1.** Depth section showing constant-time ellipse for the double-square-root equation. Source is at \( S \), receiver is at \( R \), midpoint is at \( m \), \( P = (x, y) = (m - m_0, y) \) is an image point on the ellipse, \( m_0 \) is the surface location of the image point, \( b \) is the intersection of the normal from \( P \) and the surface, \( d \) is the distance from \( P \) to \( b \), and \( h \) is half-offset.
After a log stretch velocity is unnecessary.

**RADIAL-PLANE DMO**

The methodology based on equations (4) and (5) requires a careful sampling or “binning” of \( b \) (Gardner, 1993), followed by diffraction in a radial plane. The trajectory of a single time sample \( T \) for a trace with half-offset \( h \) in the new offset space \( h_g \) is shown in Figure 2. Each fixed sample in \((m, h, t)\) space is mapped to a curve in \((m + h, h_g, t)\) space. Here, the curve lies in a radial plane, within the data volume \((m, h, t)\), defined by \( h = (h_g/T_1)/t \). It is possible to show (Gardner, 1993) that the mapping can be accomplished by \((f, k)\) migration of the radial-plane data with the “velocity” \( h_g/T_1 \). Thus, the computational requirements for velocity-independent DMO are \( O(np(n \log n)) \), where \( np \) is the number of radial planes and \( n \) is either the number of midpoints or the number of samples in each radial slice.

**CONSTANT-OFFSET DMO**

Equation (3) can be used to develop a constant-offset approach to DMO. To my knowledge, this method has not been published in a refereed journal and to some extent is poorly understood. The idea is to rewrite equation (3) in parametric form:

\[
m - m_1 = h \sin(\theta),
\]

\[
T_1 = T \cos(\theta).
\]

After a log stretch

\[
\tau = \alpha \log(T_1) + \beta,
\]

one has

\[
m - m_1 = h \sin(\theta),
\]

and

\[
\tau - \tau_1 = \alpha \log(\cos(\theta)).
\]

Thus, DMO in constant offset planes becomes a space-time convolution that can be accomplished in \((f, k)\) space by application of the operator

\[
\exp \left[ -i\pi \alpha f \left( \sqrt{1 + \frac{4h^2k^2}{\alpha^2f^2}} - 1 - \log \left( \frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{4h^2k^2}{\alpha^2f^2}} \right) \right) \right].
\]

Thus, DMO, as specified here, again has computational complexity \( O(nh(n \log n)) \), where \( nh \) is the number of offsets.

**VELOCITY-INDEPENDENT-PRESTACK IMAGING**

Regardless of how its applied, velocity-independent DMO produces a volume with normal moveout given by

\[
T^2 = T_0^2 + \frac{4((m - m_0)^2 + h^2)}{v^2}.
\]

If one estimates \( v \) from this equation, the value is relative to the time \( T_0 \) measured to the normal from \( m + b \) and not to the desired (migrated) location \( m_0 \) or time \( T_M \). Moreover, the offset term has an undesirable dependence on \( x = m - m_0 \). To see how to remove this dependence and migrate the DMO volume without knowing \( v \), let

\[
h_0^2 = (m - m_0)^2 + h^2.
\]

The volume will be focused after points on the circle with center \((m_0, 0)\) and radius \( h_0 \) are mapped to the point \((m_0, h_0)\). After focusing, the relation between \( T, T_M, h_0, \) and \( v \) is given by

\[
T^2 = T_M^2 + \frac{4h_0^2}{v^2}.
\]

At this point, velocity estimation provides an estimate of \( v \) at \( m_0 \) and time \( T_M \). Velocity-independent focusing requires that \( m_0 \) and \( h_0 \) be expressed in terms of \( m \) and \( h \). First, differentiate equation (12) with respect to \( m \) and \( h \) to get

\[
\frac{\partial T}{\partial m} = \frac{4(m - m_0)}{v^2 T},
\]

\[
\frac{\partial T}{\partial h} = \frac{4h}{v^2 T}.
\]

Then, solve for the derivative of \( h \) with respect to \( m \):

\[
-\frac{\partial h}{\partial m} = \frac{\partial T/\partial m}{\partial T/\partial h} = \frac{(m - m_0)}{h}.
\]
Finally, rewrite equation (17) as

\[ m_0 = m + h \frac{\partial h}{\partial m}, \]  

(18)

and substitute equation (18) into equation (13) so that

\[ h_0^2 = h^2 + h^2 \left( \frac{\partial h}{\partial m} \right)^2. \]  

(19)

Equations (18) and (19) define PSI. As illustrated in Figure 3, they transform constant \( T \) slices from variables \((m, h)\) to variables \((m_0, h_0)\). They accomplish their task by either summing along the circle (dotted curve) or spreading along the hyperbola (solid curve). Clearly, because there is no velocity dependence, PSI is also a velocity-independent process. As was the case for DMO, PSI can also be formulated (Gardner, 1993) as an \((f, k)\) process. In this case, the process is equivalent to \((f, k)\) (Stolt) modeling rather than migration. Note also that since the log stretch process does not distort the PSI circle in constant log stretch time slices, one can implement the entire DMO-PSI process in log stretch frequency space.

**EQUIVALENT-OFFSET MIGRATION**

Simple algebraic rationalization of equation (1) yields

\[ T^2 = T_{sh}^2 + \frac{h_e^2}{v^2}. \]  

(20)

Equation (20) is, in fact, the “equivalent-offset” formula of Bancroft and Geiger (1994) and Bancroft et al. (1995).

As described by Bancroft et al. (1995), EOM is a PSI-type process explained geometrically by Figure 4. The solid line in this figure represents the equivalent offset curve associated with the point \((m, h, T)\) in the constant time slice defined by \(T\). The amplitude at time \(T\) on a trace with midpoint \(m\) and offset \(h\) is mapped to every point on this solid curve. The dotted curve represents the quartic along which amplitudes are summed to be focused at \((m, h_e)\). Note that EOM is conceptually equivalent to PSI in that each input point at constant time \(T\) and fixed offset \(h\) corresponds to a trajectory indexed by \(T\) with the new offset, \(h_e\), varying as a function of equation (21).

Computationally, the algorithm will have characteristics similar to those of standard prestack-Kirchhoff approaches. For each fixed \(T\), one can reduce the number of calculations by carefully limiting offset sampling, but the net result is an algorithm whose order is still proportional to the product of the number of samples, the number of midpoints, and the number of offsets. Assuming that these are roughly equivalent and equal to \(n\), the operation count is approximately \(n^3\). That is, the number of calculations required to move input data is directly proportional to the number of points in the output volume, and so is of order \(n^3\) for two-dimensional problems.

The presence of \(v^2T^2\) in the denominator of equation (21) shows that EOM cannot proceed without knowledge or...
reasonable estimates of the velocity \( v \). Clearly, the accuracy with which \( v \) is known has a direct and immediate bearing on the accuracy of the final migration. With the exception of the fact that residual NMO can improve the final image, one can expect the result of EOM to be basically similar to a good-quality standard Kirchhoff migration.

To fully understand the similarities between EOM and DMO-PSI, one can attempt to follow the logic of equations (15), (17), (18), and (19). Repeating the exercise with equation (20) replacing equation (12) and \( m_0 = m_r \) yields

\[
\frac{\partial T}{\partial m} = 4(m - m_r) \left( 1 - \frac{4h^2}{v^2T^2} \right) \left( v^2T - \frac{4h^2x^2}{v^2T^3} \right)^{-1},
\]

Equation (24) can be solved for \( T \) in terms of \( m, h, T \), and \( m_0 = m_r \), by summing over the dotted curves. In the spirit of EOM the solid curves or by summing over the dotted curves. In the limit, the solid curves are the PSI curves. This separability also allows one to perform the imaging step on constant time slices by diffracting energy over appropriately defined curves. The PSI curves are much simpler than those for EOM, and so are significantly easier to implement.

Theoretically, EOM is equivalent to PSI when the input data have been DMO-corrected. In that case, the equivalent-offset formula for offset \( h_e \) can be substituted into equation (21) to specify the equivalent-offset quartic, respectively. The dotted curve in Figure 4 schematically represents the locus of points \( \hat{e} \) which map to \( (m_0, h_e, T) \) or \( (m_0, h_0) \). The separating curves are circles rather than quartics. There are many alternative formulations of DMO-PSI, but the fact that the entire process can be completed in the frequency-wavenumber domain implies the existence of a very fast algorithm that is computationally superior to diffraction stacking.

Separation of prestack-constant-velocity migration into two steps (DMO and PSI) has an advantage over EOM in that it allows free choice of the DMO algorithm. The use of more sophisticated DMO techniques prior to PSI (e.g., Alfaraj and Larner, 1991, 1992; Anderson and Tsvankin, 1994; Anderson et al., 1994; Miller and Burridge, 1989; Liner, 1990, 1991) may improve both image quality and velocity estimates. Inclusion of reasonable isotropic velocity variations is possible in DMO-PSI. As anisotropic techniques mature, it will also be possible to include anisotropic velocity variations in the imaging equation. This separability also allows one to apply a 3-D DMO-PSI sequence followed by inverse 2-D DMO to regularize irregular data sets and use 2-D prestack-depth migration to achieve a 3-D result (Canning and Gardner, 1993).

Velocity independence is definitely a powerful feature of any prestack technique. The inherent dependence of EOM on velocity estimates reduces its potential as a competitive alternative to DMO-PSI.

Acknowledgments

Jon Claerbout gets the award for providing the space, resources, and a great environment to do work in. Thanks go to him, S. Fomel, and M. Prucha at the Stanford Exploration Project, and K. Larner at the Colorado School of Mines for keeping my wording, spelling, and references honest. W. Symes at Rice, G. Neale in Tulsa, and N. Bleistein at the Colorado School of Mines are also acknowledged for supporting and encouraging completion of this work. P. Fowler showed me the relationship between the various techniques mentioned in the introduction. G. Gardner and T. Alkalifah provided excellent suggestions in review.

References


