Analysis of azimuthal anisotropy in coal-scale 3D seismic reflection

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Abstract

Azimuthal anisotropy in seismic velocity is a well known phenomenon, although it is often ignored in conventional reflection processing. Such anisotropy can result from sub-vertical fracturing often related to orientation of horizontal stress. It can also be an artefact of dipping interfaces in an isotropic system.

Some understanding of the likely contributions can be obtained from simple numerical models. If dips are less than 10 degrees, then spurious azimuthal effects are likely to be less than 2%. If reasonable values of Thompson parameters are assumed, true velocity anisotropy can be well in excess of 10%. If this is ignored in the NMO process, then smearing is expected, particularly at the coal scale where dominant frequencies in excess of 100 Hz can be expected.

A robust algorithm has been implemented to invert reflection travel times in terms of an azimuthally anisotropic velocity model. The inversion yields the fast azimuth (major axis of best-fit ellipse) and the degree of anisotropy (derived from the ellipticity). The algorithm performs well on a high-fold 3D P-wave survey from the Bowen Basin. A very consistent pattern of azimuthal anisotropy is observed across the survey area, with largest magnitudes around 9%. The observations cannot be explained in terms of known dip and are believed to represent true azimuthal effects.

When the detected anisotropy is allowed for in the NMO correction, CMP gathers show a clear improvement in event alignment. However, this improvement does not flow through to the stack. For these shallow data, factors such as NMO stretch and statics are significant and it is possible that such effects are overriding the improvement from anisotropic NMO.

A preliminary analysis of an associated 3D converted-wave (PS) survey has detected anisotropy parameters (magnitude, direction) which are broadly consistent with the P-wave results. The analysis was done using the same P-wave inversion algorithm. The validity of this approach and the true physical significance of the results (in terms of P-wave and S-wave anisotropy) requires further research.

Overall, the inversion technique implemented here appears to provide a robust mechanism for characterising azimuthal anisotropy in P and PS data, at the shallow-coal scale.
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Chapter 1

Introduction

1.1 Aims of Thesis

This thesis implements an inversion algorithm for estimation of azimuthal velocity anisotropy in 3D seismic reflection data. The process is based on the generalised 3D normal moveout equation developed by Grechka and Tsvankin (1998). The algorithm will be tested on synthetic data and a 3D coal-scale set.

There are two practical objectives. Firstly we attempt to extract anisotropic information possibly relating to fracturing. Secondly we examine whether stacking quality can be improved for shallow data if anisotropy is allowed for.

1.2 Historical Background

Thomsen (2002) defines seismic anisotropy as the large-scale manifestation of ordered, small-scale heterogeneity. These heterogeneities in a medium cause the P-wave velocity to become directionally dependent. If anisotropy is not corrected for during processing it can produce misalignment of reflectors and interference in stacking.

Historically seismic anisotropy was considered a theoretical branch of seismology and was not accounted for in seismic processing. However, it has become an increasingly important subject in current practical research. Anisotropy was introduced to the field of geophysics in the late 19th century by Maurice Rudzki, the first official professor of geophysics. Rudzki was a major advocate that all rocks were anisotropic and published several papers trying to determine the wavefront for transverse isotropic (TI) and orthorhombic media, although he was initially limited by numerical difficulties (Helbig et al., 2000). After Rudzki’s death in
1916 research on seismic anisotropy was very limited until the mid-to-late 20th century.

Postma (1955) and Helbig (1956) independently laid out the founding theory for seismic anisotropy. Postma described wave propagation in a stratified media by deriving the wave equation from the stress-strain relationship of transversely isotropic media and the equations of motion. He showed that there were three body wave velocities that were dependent on the direction of propagation. Krey and Helbig (1956) showed that layers with constant $v_P/v_S$ ratio can be considered isotropic for reflections with small dip. This result halted further progress on anisotropy research, based on the argument that the anisotropic effect could be ignored as long as surveys had restricted offset.

Thomsen introduced the dimensionless parameters $\epsilon, \delta$ and $\gamma$ as a convenient measure of anisotropy in his pivotal paper “Weak elastic anisotropy” (1986). These parameters replaced the elastic moduli to describe velocity variations with direction which were reduced to a simple expression under the condition of weak anisotropy. Weak anisotropy was defined as the case where the anisotropic parameters $\epsilon, \delta, \gamma << 1$. It was determined that most sedimentary rocks have been shown to be weakly anisotropic ($< |0.2|$) even if the constituent minerals exhibit strong anisotropy. The Thomsen parameter $\delta$ was found to significantly control the anisotropic nature of the media in most geophysical situations. The value of $\delta$ can be both negative and positive. Thomsen notation has been extended to more complex symmetries; orthorhombic (Tsvankin, 1997a), monoclinic (Grechka et al., 2000) and triclinic (Mensch and Rasolofosaon, 1997).

Alkhalifah and Tsvankin (1995) introduced the anellipticity coefficient, $\eta$ which describes the deviation from a hyperbolic moveout. It was found for VTI media that the P-wave NMO velocity and anellipticity can be used to perform time processing steps including NMO correction, dip moveout and prestack and poststack migration (Tsvankin, 1997b). This was extended to orthorhombic media with the inclusion of additional anellipticity coefficients (Grechka and Tsvankin, 1999b).

With isotropic media, the reflection moveout can be approximated by the equation $t^2_x \approx t^2_0 + \frac{x^2}{v_{nmo}^2}$ which is of the form of a hyperbola. For anisotropic media, this hyperbolic approximation is only appropriate for short offsets. To accommodate larger offsets, a general analytical nonhyperbolic approximation was determined. Tsvankin and Thomsen (1994) tested the fourth-order Taylor form and found that at increasing offsets it was numerically inaccurate. Their general moveout equation was based on the exact Taylor
series coefficients and has the form of the weak anisotropy approximation. Therefore for P-wave data, the proposed general moveout equation is suitable for large offsets and strongly anisotropic media.

A 3D NMO equation for anisotropic media was developed by Grechka and Tsvankin (1998). The equation is the basis for the inversion performed in this thesis. The NMO velocity equation is for a model with arbitrary anisotropy and inhomogeneity. It can model P-wave or S-wave velocities. The velocity fits an elliptical form which is a function of ray azimuth, and the azimuth of the geological features causing anisotropy (e.g. fractures). The derivation of the 3D NMO equation is analysed in detail in Chapter 4.

1.3 Thesis Outline

This thesis will focus on seismic anisotropy and the 3D normal moveout of data.

Chapter 2 will highlight the important background theory for seismic anisotropy. The Thomsen parameters for VTI and HTI media will be examined. The NMO velocity for an arbitrary anisotropic media can be described by an ellipse function. A complication in determining whether the data has true azimuthal anisotropy is that the form of the NMO velocity for dipping reflectors is also an ellipse. If the approximate dip of an area is known, it can be determined whether the strength of anisotropy exhibited is most likely caused by dipping reflectors or true azimuthal anisotropy.

Levin (1971) gives the apparent velocity from a dipping reflector for homogeneous isotropic media. The strength of anisotropy caused by these dipping reflectors will be modelled in Chapter 3. Several different materials and dip angles will be used to estimate the percentage of anisotropy expected. This will be plotted and compared to the NMO velocity equation for horizontal reflectors which was presented for HTI media by Tsvankin (1997b). The NMO expression is described in terms of Thomsen parameters.

To test for anisotropy in data, a synthetic model must be constructed. Chapter 4 runs the inversion code on several grid arrays to simulate one shot point. This will involve isotropic and anisotropic earth models for pure P-wave data.

The data in this thesis were collected from a 3D-3C survey in the Bowen Basin. Chapter 5 will review past analyses on the same data. The processing stages for the data will be outlined and picking methods will be compared. Two areas in the survey were chosen for
analysis. The normal moveout correction will be applied to a receiver line to visualise the reflector along the extent of the survey.

A converted-wave data set in the survey area was available for evaluation. Preliminary inversion results will be compared with the P-wave data in Chapter 6.

Chapter 7 will summarise the main findings and conclusions reached in the thesis. A recommendation for future studies will be given.
Theory of Seismic Anisotropy

Anisotropy occurs when properties of the media are directionally dependent. Seismic anisotropy can be generated by:

- preferred orientation of mineral grains,
- preferred orientation of clastics,
- orientation of stress fields and fractures.

This suggests that it might be revealed by P or S waves travelling, or polarised, in different directions.

There are two main types of anisotropy that are important when investigating sedimentary rocks (Chopra and Castagna, 2014). Layer anisotropy develops during deposition and it is governed by the orientation of clastics. The factors include grain size, ordering of grains and sediment type. Intrinsic rock anisotropy deals with the orientation of mineral grains and is determined at the microscopic scale. Deformation and compaction from stress can cause grain alignment. Stress and fracture anisotropy are special cases of intrinsic anisotropy which arise from the orientation of stress fields and subsurface fractures. Anisotropic symmetries such as vertical transverse isotropy and horizontal transverse isotropy (described in Chapter 2.1) are respectively the results of layer and intrinsic anisotropy.

Apparent azimuthal anisotropy can be the result of true azimuthal anisotropy, but also other factors including PS conversion, dipping reflectors, attenuation and density variations (Anderson, 1989). The azimuthal dependence of seismic waves is evident in shear-wave splitting, moveout velocities for P and converted-wave data and amplitudes of the waves.
Chapter 2. Theory of Seismic Anisotropy

This thesis is concerned with true azimuthal anisotropy. It is often the result of fractures in the earth but can also form from sedimentary fabrics and stress (Thomsen, 2002). This chapter will examine the basic theory for seismic anisotropy consisting of model symmetries and their relevant Thomsen parameters. The normal moveout equation that can be expressed as an elliptical curve will be considered for general anisotropy and dipping reflectors. The NMO theory will be the basis for Chapters 4 and 5.

2.1 Anisotropic Symmetries

Although this thesis focuses on azimuthal anisotropy, we will start with a more general overview. This section on anisotropic symmetries comes from Chapter 1 in Tsvankin (2012).

Hooke’s Law states that given the stress acting on a solid is less than its elastic limit then the stress will be directly proportional to the strain produced. The stress-strain relationship for elastic isotropic media satisfies Hooke’s law via

\[ P_{ik} = \lambda \delta_{ik} + 2\mu e_{ik} \]

where \( \delta \) is the Kronecker delta tensor, \( \lambda \) and \( \mu \) are Lame’s constants.

The six independent stress components are:

\[
\begin{align*}
P_{11} &= \lambda e_{11} + \lambda e_{22} + \lambda e_{33} + 2\mu e_{11} \\
P_{22} &= \lambda e_{11} + \lambda e_{22} + \lambda e_{33} + 2\mu e_{22} \\
P_{33} &= \lambda e_{11} + \lambda e_{22} + \lambda e_{33} + 2\mu e_{33} \\
P_{23} &= 2\mu e_{23} \\
P_{13} &= 2\mu e_{13} \\
P_{12} &= 2\mu e_{12}
\end{align*}
\]
These stress-strain relationships can be expressed using Voigt matrix notation.

\[
\begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{bmatrix}
= 
\begin{bmatrix}
\lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{bmatrix}
\begin{bmatrix}
e_{11} \\
e_{22} \\
e_{33} \\
e_{23} \\
e_{13} \\
e_{12}
\end{bmatrix}
\]  
(2.1)

where \(C_{11} = C_{33} = \lambda + 2\mu, C_{44} = C_{66} = \mu\) and \(C_{13} = \lambda\)

In reality the stress-strain relationships are more complex. The general model is triclinic symmetry which involves 21 stiffness coefficients and has three symmetry planes at arbitrary angles. Other symmetries can be considered special cases of triclinic symmetry, which has the following Voigt matrix equation

\[
\begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{bmatrix}
= 
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & C_{44} & C_{25} & C_{26} \\
C_{13} & C_{23} & C_{33} & C_{55} & C_{35} & C_{36} \\
C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\
C_{15} & C_{25} & C_{35} & C_{55} & C_{56} & C_{56} \\
C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}
\end{bmatrix}
\begin{bmatrix}
e_{11} \\
e_{22} \\
e_{33} \\
e_{23} \\
e_{13} \\
e_{12}
\end{bmatrix}
\]  
(2.2)

Monoclinic symmetry has 12 stiffness coefficients with three symmetry planes. One plane is at an arbitrary angle while the other two planes are at 90 degrees. This models a rock which is affected by two non-orthogonal sets of fractures.

\[
\begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{bmatrix}
= 
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{13} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
e_{11} \\
e_{22} \\
e_{33} \\
e_{23} \\
e_{13} \\
e_{12}
\end{bmatrix}
\]  
(2.3)

Orthorhombic symmetry has 9 stiffness coefficients and has three symmetry planes each separated by 90 degrees. This models a thin-bed layered earth with vertical fractures and
can be considered as a realistic symmetry for many geophysical problems.

$$
\begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{bmatrix}
= 
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
e_{11} \\
e_{22} \\
e_{33} \\
2e_{23} \\
2e_{13} \\
2e_{12}
\end{bmatrix}
$$

(2.4)

The simplest practical symmetry is transverse isotropy or TI. It is widely used in geophysical studies on anisotropy due to its acceptable approximation of realistic geology models. The general form of TI is tilted transverse isotropy (TTI) and there are two special cases - when the symmetry axis is vertical and when the symmetry axis is horizontal. The symmetry matrix written below (Equation 2.5) shows vertical transverse isotropy or VTI. This is a layered earth model without fractures which can be described by 5 stiffness coefficients. It can have an infinite number of symmetry planes and has a vertical symmetry axis.

$$
\begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{bmatrix}
= 
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{11} - 2C_{66} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
e_{11} \\
e_{22} \\
e_{33} \\
2e_{23} \\
2e_{13} \\
2e_{12}
\end{bmatrix}
$$

(2.5)

If the symmetry axis is horizontal it is called horizontal transverse isotropy or HTI and can be geologically represented as single layer with vertical fractures.

### 2.2 Thomsen Parameters

The Thomsen parameters were described by Leon Thomsen in his pivotal paper “Weak elastic anisotropy” (1986). These parameters were introduced as a convenient measure of anisotropy and are dimensionless functions of the stiffness coefficients.

Tsvankin (1997a) replaced the stiffness coefficients with two vertical velocities (P and S wave) and new Thomsen parameters that describe orthorhombic media. Because VTI and HTI are degenerative special cases of orthorhombic symmetry, we will present these extended parameters from that paper.
\[ \epsilon^{(1)} = \frac{C_{22} - C_{33}}{2C_{33}} \]
\[ \delta^{(1)} = \frac{(C_{23} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})} \]
\[ \gamma^{(1)} = \frac{C_{66} - C_{55}}{2C_{55}} \]
\[ \epsilon^{(2)} = \frac{C_{11} - C_{33}}{2C_{33}} \]
\[ \delta^{(2)} = \frac{(C_{13} + C_{55})^2 - (C_{33} - C_{55})^2}{2C_{33}(C_{33} - C_{55})} \]
\[ \gamma^{(2)} = \frac{C_{66} - C_{44}}{2C_{44}} \]
\[ \delta^{(3)} = \frac{(C_{12} + C_{66})^2 - (C_{11} - C_{66})^2}{2C_{11}(C_{11} - C_{66})} \]

The superscripts (1), (2) and (3) represent the parameter in the \([x_2, x_3]\), \([x_1, x_3]\) and \([x_1, x_2]\) plane respectively.

### 2.2.1 Vertical Transverse Isotropy

For VTI media, the Thomsen parameters can be written as

\[ \epsilon^{(1)} = \epsilon^{(2)} = \epsilon \]
\[ \delta^{(1)} = \delta^{(2)} = \delta \]
\[ \gamma^{(1)} = \gamma^{(2)} = \gamma \]
\[ \delta^{(3)} = 0 \]

The parameters without superscripts are those described initially by Thomsen (Equations 2.6 - 2.8).

\[ \epsilon = \frac{C_{11} - C_{33}}{2C_{33}} \] (2.6)
\[ \delta = \frac{(C_{13} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})} \] (2.7)
\[ \gamma = \frac{C_{66} - C_{44}}{2C_{44}} \] (2.8)
Note that for isotropic media, the wave equation gives two body wave solutions

\[ v_P = \sqrt{\frac{\lambda + 2\mu}{\rho}} \text{ and } v_S = \sqrt{\frac{\mu}{\rho}} \]

Assuming vertical transverse isotropy with a case of weak anisotropy, there are three body wave solutions as shown in Equations 2.9 - 2.11.

\[ v_P(\theta) \approx v_{P0}[1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta] \quad (2.9) \]
\[ v_{S\perp}(\theta) \approx v_{S0}[1 + (\frac{v_{P0}}{v_{S0}})^2(\epsilon - \delta) \sin^2 \theta \cos^2 \theta] \quad (2.10) \]
\[ v_{S\parallel}(\theta) \approx v_{S0}[1 + \gamma \sin^2 \theta] \quad (2.11) \]

where \( v_{P0} = \sqrt{\frac{C_{33}}{\rho}} \) and \( v_{S0} = \sqrt{\frac{C_{44}}{\rho}} \).

To determine the meaning of \( v_{P0}, v_{S0}, \epsilon, \delta \) and \( \gamma \), consider the body wave solutions for vertical (\( \theta = 0 \)) and horizontal (\( \theta = 90 \)) propagation.

\[ v_P(0) = v_{P0} \quad v_P(90) = v_{P0}[1 + \epsilon] \]
\[ v_{S\perp}(0) = v_{S0} \quad v_{S\perp}(90) = v_{S0} \]
\[ v_{S\parallel}(0) = v_{S0} \quad v_{S\parallel}(90) = v_{S0}[1 + \gamma] \quad (2.12) \]

The velocities \( v_{P0} \) and \( v_{S0} \) are the velocities of vertically travelling P and S waves respectively. For horizontal propagation there are now two different S-wave velocities for the polarisation parallel and perpendicular to layering.

By rearranging Equations 2.12, the parameters \( \epsilon \) and \( \gamma \) can be shown to be fractional differences between the vertical and horizontal velocities for P and S-waves respectively.

\[ \epsilon = \frac{v_{P}(90) - v_{P0}}{v_{P0}} \]
\[ \gamma = \frac{v_{S\parallel}(90) - v_{S0}}{v_{S0}} \]

It can be concluded that positive \( \epsilon \) and \( \gamma \) values indicate that the P-wave and S-wave velocity of the media is faster in the horizontal direction. It should be noted that all of the
Thomsen parameters can be both negative or positive, however $\epsilon$ and $\gamma$ are generally positive.

### 2.2.2 Horizontal Transverse Isotropy

In the previous subsection, vertical transverse isotropy was examined in detail.

A HTI system can be represented in Voigt matrix notation as

$$
\begin{bmatrix}
P_{11} \\
P_{22} \\
P_{33} \\
P_{23} \\
P_{13} \\
P_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{13} & C_{13} & 0 & 0 & 0 \\
C_{13} & C_{33} & C_{33} - 2C_{44} & 0 & 0 & 0 \\
C_{13} & C_{33} - 2C_{44} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{55}
\end{bmatrix}
\begin{bmatrix}
e_{11} \\
e_{22} \\
e_{33} \\
2e_{23} \\
2e_{13} \\
2e_{12}
\end{bmatrix}
$$

Horizontal transverse isotropy or HTI can be visualised as a VTI system that is rotated 90 degrees. The extended Thomsen parameters are

$$
\begin{align*}
\epsilon^{(1)} &= 0 \\
\delta^{(1)} &= 0 \\
\gamma^{(1)} &= 0 \\
\epsilon^{(2)} &= \epsilon^{(V)} \\
\delta^{(2)} &= \delta^{(V)} \\
\gamma^{(2)} &= \gamma^{(V)}
\end{align*}
$$

The Thomsen parameters for HTI media were distinguished from VTI parameters with a superscript (V) by Tsvankin (1997b). Equations 2.14 - 2.16 shows these parameters in terms
Chapter 2. Theory of Seismic Anisotropy

of stiffness coefficients.

\[
\epsilon(V) = \frac{C_{11}^{(V)} - C_{33}^{(V)}}{2C_{33}^{(V)}} \tag{2.14}
\]

\[
\delta(V) = \frac{(C_{13}^{(V)} + C_{55}^{(V)})^2 - (C_{33}^{(V)} - C_{55}^{(V)})^2}{2C_{33}^{(V)}(C_{33}^{(V)} - C_{55}^{(V)})} \tag{2.15}
\]

\[
\gamma(V) = \frac{C_{66}^{(V)} - C_{44}^{(V)}}{2C_{44}^{(V)}} \tag{2.16}
\]

It should be noted that the stiffness coefficients for different symmetries are not identical. To determine a relationship between HTI and VTI, the rotated coordinate system needs to be accounted for. The relations for the stiffness coefficients between HTI and VTI media are:

\[C_{11}^{(V)} = C_{33}^{(V)} = C_{11}, \quad C_{13}^{(V)} = C_{13}, \quad C_{55}^{(V)} = C_{55}, \quad C_{44}^{(V)} = C_{66} \quad \text{and} \quad C_{66}^{(V)} = C_{44}^{(V)}.
\]

Thus the HTI Thomsen parameters can be written in terms of the VTI parameters as

\[
\epsilon(V) = \frac{\epsilon}{1 + 2\epsilon}
\]

\[
\delta(V) = \frac{\delta - 2\epsilon[1 + \epsilon/f]}{[1 + 2\epsilon][1 + 2\epsilon/f]}
\]

\[
\gamma(V) = -\frac{\gamma}{1 + 2\gamma}
\]

where \( f = 1 - \left[\frac{V_{S0}}{V_{P0}}\right]^2 \).

2.3 Normal Moveout Equations

The reflection time for a homogeneous single layered earth is

\[
t_x^2 = t_0^2 + \frac{x^2}{v^2} \tag{2.17}
\]

where \( t_0 \) is the zero offset travel time, \( x \) is the offset and \( v \) is the NMO velocity.

The NMO velocity for azimuthally anisotropic medium can be described by an ellipse function. Determining whether anisotropy in data occurs is complicated by the fact that for dipping reflectors the form of the NMO velocity function is also an ellipse.
Chapter 2. Theory of Seismic Anisotropy

2.3.1 Anisotropic Media

When the medium is inhomogeneous and exhibits arbitrary anisotropy, the NMO velocity equation to be used is

\[
\frac{1}{v_{nmo}^2(\alpha)} = W_{11}\cos^2(\alpha) + 2W_{12}\sin(\alpha)\cos(\alpha) + W_{22}\sin^2(\alpha) \tag{2.18}
\]

\[
= \lambda_1\cos^2(\alpha - \beta) + \lambda_2\sin^2(\alpha - \beta) \tag{2.19}
\]

where \(\beta\) is the rotation angle, \(\alpha\) is the source-receiver azimuth and \(\lambda_1, \lambda_2\) are eigenvalues of the matrix \(W\). The angle \(\beta\) is found with respect to the symmetry axis which can be parallel to the major or minor velocity axes (Tsvankin and Grechka, 2011).

Different anisotropic models (e.g. HTI, orthorhombic, monoclinic) can result in elliptical azimuthal variation of the NMO velocity (Al-Dajani, 2002). This equation can be used to calculate the P-wave NMO velocity for arbitrary anisotropic media with horizontal or dipping reflectors (Grechka and Tsvankin, 1998). However, it cannot distinguish between different anisotropic models.

It was proved that the NMO curve can be represented as an ellipse when \(\lambda_1, \lambda_2 > 0\). These eigenvalues are positive for most geological situations. This equation fails for extreme cases where the CMP reflection time decreases with offset, i.e. when \(\lambda_1\) or \(\lambda_2\) is negative. In some azimuths this results in \(v_{nmo}^2 < 0\) which cannot be modelled by an ellipse. Thus the moveout approximation will not be accurate. If \(\lambda_1\) or \(\lambda_2\) is zero, the NMO ellipse reduces into two straight lines parallel to the direction \(v_{nmo} = \infty\).

In the formulation of this equation, no specific assumptions were made. Thus it is valid for smooth reflectors. The travel time was approximated as a Taylor series expansion up to the second order. This is known to break down in strong lateral velocity variation. It was also assumed that near CMP points, the travel time exists at any azimuth. This will fail in shadow zones.

For HTI media, the symmetry axis is in the direction of the minor axis and \(\beta\) is therefore the angle from the slow velocity axis (Jenner, 2010). The NMO velocity equation can be expressed for HTI media as

\[
\frac{1}{v_{nmo}^2(\alpha)} = \frac{1}{v_{slow}^2}\cos^2(\alpha - \beta_{slow}) + \frac{1}{v_{fast}^2}\sin^2(\alpha - \beta_{slow}) \tag{2.20}
\]
Contreras et al. adapted the anisotropic NMO velocity equation (Equation 2.18) to express the P-wave NMO velocity in terms of the Thomsen parameter \( \delta(V) \), azimuth, rotation angle and vertical velocity.

\[
v_{nmo}^2(\alpha) = \frac{\cos^2(\alpha - \beta)}{v_{P,nmo}^2} + \frac{\sin^2(\alpha - \beta)}{v_{P0}^2}
\]

(2.21)

where \( v_{P,nmo} = v_{P0}\sqrt{1 + 2\delta(V)} \). This NMO equation can be used to determine the Thomsen parameters given that the vertical velocity has been determined using additional information.

### 2.3.2 Dipping Reflectors

Equation 2.22 gives the apparent velocity from a dipping reflector in a homogeneous isotropic media. It was determined by Levin (1971) and is a function of azimuth (\( \alpha \)) and dip (\( \phi \)).

\[
v_{nmo}^2(\alpha, \phi) = \frac{v^2}{1 - \cos^2 \alpha \sin^2 \phi}
\]

(2.22)

Tsvankin (1997b) showed that this equation can be expressed in the form of an ellipse (Equation 2.23) where the major and minor axes correspond to the dip and strike directions.

\[
\frac{1}{v_{nmo}^2(\alpha, \phi)} = \frac{\sin^2 \phi}{v^2} + \frac{\cos^2 \alpha}{(v/\cos \phi)^2}
\]

(2.23)

Grechka and Tsvankin (1998) showed that dipping reflectors in VTI media will also produce a NMO ellipse. The NMO equation can be expressed as

\[
\frac{1}{v_{nmo}^2(\alpha, \phi)} = \frac{\cos^2 \alpha}{v_{nmo}^2(0, \phi)} + \frac{\sin^2 \alpha}{v_{nmo}^2(\pi/2, \phi)}
\]

(2.24)

where \( v_{nmo}^2(\alpha = 0, \phi) \) is the velocity in the dip line and \( v_{nmo}^2(\alpha = \pi/2, \phi) \) is the velocity in the strike line.
Elliptical dependence for NMO velocity is often considered a manifestation of azimuthal anisotropy. However a dipping reflector will also give a velocity ellipse and therefore can be mistaken for azimuthal anisotropy. Chapter 3 will examine the percent of anisotropy (ellipticity) certain dips will produce to determine whether azimuthal anisotropy can be distinguished from dipping reflectors.

### 2.4 Methodologies for Azimuthal Anisotropy

There are many methods that have been proposed to help extract information from data exhibiting anisotropy. Information including the Thomsen parameters, normal moveout velocity, percentage anisotropy and fracture orientation are of great interest. Allowing for the effects of anisotropy in the data can allow for more accurate processing. Several methods that are used with azimuthally anisotropic data will be examined in this section. Not all these methods are exclusive to azimuthal variation and can be applied to other forms of anisotropy.

Grechka and Tsvankin (1998) presented a NMO equation for horizontal and dipping reflectors in anisotropic media. It has been extensively used (Jenner 2010, Grechka and Tsvankin 1999a, Grechka et al. 2000, Contreras et al. 1999, Alkhalifah and Tsvankin 1995) for media of differing symmetry types to invert for the NMO velocity. The algorithm is used in this thesis and discussed in detail in Chapter 4.

The elliptical NMO equation described by Grechka and Tsvankin (1998) was expressed in terms of Thomsen parameters. This equation is the basis for many inversion algorithms with azimuthally variant velocities. The NMO velocity in HTI media is a function of two Thomsen parameters; \( \delta(V) \) and \( \epsilon(V) \) where the superscript represents equivalent VTI Thomsen parameters. Grechka and Tsvankin (1999b) represented the P-wave NMO ellipse as a function of the symmetry orientation (\( \beta \)), vertical velocity, and the relevant Thomsen parameters.

The anellipticity, \( \eta \) is the deviation from a hyperbolic moveout at long-offsets. It can be determined from an inversion procedure (Tsvankin, 1997b) that requires two NMO ellipses from reflectors of different dips or azimuths. The algorithm uses elements from the slowness matrix as input data to determine the parameter \( \eta(V) \). The P-wave NMO velocity and anellipticity coefficient are important for time imaging while the vertical velocity and anisotropy coefficients \( \epsilon \) and \( \delta \) are required for depth imaging (Grechka and Tsvankin,
Liu et al. (2014) were able to determine the Thomsen parameters from VSP data with a high level of accuracy. The vertical reflection time and its velocity were found from the zero-offset VSP. For a range of $\delta$ values the reflection travel times were calculated and compared to the observed travel times. The $\delta$ which gave the minimum difference in times was chosen. The parameter $\epsilon$ was determined similarly by using Equation 2.25 to calculate velocity and then first arrival travel times.

$$v_P(\theta) = V_{P0}(1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta)$$

(2.25)

The parameters obtained were used to create an anisotropic depth velocity model that was used for prestack migration. Using this model instead of assuming an isotropic and homogeneous media improved the imaging of the depth migration and minimised errors.

It has been shown by Grechka and Tsvankin that the velocity of a dipping reflector can have an elliptical form thus complicating anisotropy determination. Kuehnel and Li (1996) constructed a separation approach based on the travel time equation to remove the effects of dip. The separation approach can be simply stated as a medium with anisotropy and structure is equal to the anisotropy of the medium plus its isotropic structure.

The travel time equations were calculated for a weakly anisotropic dipping medium with components separated into dependent and independent parts. By comparing different materials and dip angles, it was found that the anisotropic influence was of an order less than the dip. In theory, the dip angle and its influence could then be calculated and removed from the data. The separation procedure was examined by having the travel times for a reference model without structure compared to the travel times of models with dips of 10-20\% removed from the data. If the method worked perfectly, the travel times against distance should be identical. The separation approach worked accurately for short offset (incident angles less than 30 degrees) and is limited by the degree of anisotropy. The method can be applied to data with 15-20\% anisotropy and up to 20 degrees dip.

Media that are significantly anelliptic result in non-hyperbolic moveout curves (Sayers, 1995) that complicate velocity analysis. Sayers and Ebrom (1997) presented a method to perform velocity analysis on azimuthally varying media with anellipticity. The seismic traveltimes were found by considering the ray geometry in an azimuthally anisotropic medium. Information on the orientation of symmetry planes is not required.
A function $r(\theta, \phi)$ was introduced as the inverse square of the ray velocity. This function can be written as a linear combination of spherical harmonics and can be simplified if the symmetry of the medium is known. Using this function, the travel time equation can then be expressed as Equation 2.26 where $v_v$, $v_{nmo}$ and $A$ are the vertical velocity, NMO velocity and anellipiticity of the medium. Tsvankin and Thomsen (1994) show that the parameters can be calculated using the elastics constants under the assumption of weak anisotropy. Weak anisotropy is not assumed and thus the coefficients are not related to the elastic constants and are determined by fitting the travel time equation to the data using a least squares method.

\[ t^2(x) = \frac{z^2}{v_v^2} + \frac{x^2}{v_{nmo}^2} - \frac{Ax^4}{x^2 + z^2} \]  

(2.26)
Modelling Azimuthal Anisotropy

Seismic anisotropy manifests itself in data by causing processed images to have misplaced and out of focus reflectors. These effects can be removed by accounting for azimuthal anisotropy. However, as discussed in Chapter 2 dipping reflectors can also show elliptical variation of velocity. The aim of this chapter is to model and compare anisotropic effects for a true anisotropic medium, and an isotropic medium with a dipping reflector.

3.1 Anisotropic Medium

The NMO equation (Equation 3.1) for HTI media can be expressed in terms of azimuth, vertical velocity and rotation angle. Contreras et al. (1999) adapted the anisotropic NMO velocity equation to expressed the P-wave velocity and to include the Thomsen parameter \( \delta^{(V)} \).

\[
\nu_{nmo}^2(\alpha) = \frac{\cos^2(\alpha - \beta)}{\nu_{P,nmo}^2} + \frac{\sin^2(\alpha - \beta)}{\nu_{P0}^2} \tag{3.1}
\]

where \( \nu_{P,nmo} = \nu_{P0} \sqrt{1 + 2\delta^{(V)}} \).

Thomsen (1986) presented a table of data on the anisotropy of sedimentary rocks, which was used to calculate suitable values for the parameter \( \delta^{(V)} \) using Equation 3.2.

\[
\delta^{(V)} = \frac{\delta - 2\epsilon[1 + \epsilon/f]}{[1 + 2\epsilon][1 + 2\epsilon/f]} \tag{3.2}
\]

where \( f = 1 - \left[ \frac{V_{S0}}{V_{P0}} \right] \). This relation was developed by Tsvankin (1997b).

Table 3.1 shows the well-known and documented values for several VTI media to calculate the Thomsen parameter for HTI media. It was determined that the HTI parameter \( \delta^{(V)} \) has
a similar range to the general $\delta$ but were largely negative. In practice, we should expect $\delta^{(V)}$ to be between 0 and -0.2. Tsvankin (1997b) determined that for weak anisotropy, $\delta^{(V)} << 1$ and is typically negative for HTI media while for VTI media $\delta^{(V)}$ may be negative or positive.

Table 3.1: Anisotropy parameters in selected sedimentary rocks taken from Thomsen (1986).

<table>
<thead>
<tr>
<th>Sample</th>
<th>$v_P$ (m/s)</th>
<th>$v_S$ (m/s)</th>
<th>$\epsilon$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$\delta^{(V)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandera sandstone</td>
<td>3810</td>
<td>2368</td>
<td>0.030</td>
<td>0.030</td>
<td>0.045</td>
<td>-0.016</td>
</tr>
<tr>
<td>Dog Creek shale</td>
<td>1875</td>
<td>826</td>
<td>0.225</td>
<td>0.345</td>
<td>0.100</td>
<td>-0.203</td>
</tr>
<tr>
<td>Ft. Union siltstone</td>
<td>4877</td>
<td>2941</td>
<td>0.045</td>
<td>0.040</td>
<td>-0.045</td>
<td>-0.109</td>
</tr>
<tr>
<td>Green River shale</td>
<td>3292</td>
<td>1768</td>
<td>0.195</td>
<td>0.180</td>
<td>-0.220</td>
<td>-0.302</td>
</tr>
<tr>
<td>Mesaverde mud shale</td>
<td>4529</td>
<td>2703</td>
<td>0.034</td>
<td>0.046</td>
<td>0.211</td>
<td>0.110</td>
</tr>
<tr>
<td>Mesaverde sandstone</td>
<td>3962</td>
<td>2926</td>
<td>0.055</td>
<td>0.041</td>
<td>-0.089</td>
<td>-0.141</td>
</tr>
<tr>
<td>Mesaverde shale</td>
<td>4359</td>
<td>3048</td>
<td>0.172</td>
<td>0.157</td>
<td>0.000</td>
<td>-0.188</td>
</tr>
<tr>
<td>Oil shale</td>
<td>4231</td>
<td>2539</td>
<td>0.200</td>
<td>0.510</td>
<td>0.100</td>
<td>-0.179</td>
</tr>
<tr>
<td>Pierre shale</td>
<td>2202</td>
<td>969</td>
<td>0.015</td>
<td>0.030</td>
<td>0.060</td>
<td>0.027</td>
</tr>
<tr>
<td>Taylor sandstone</td>
<td>3368</td>
<td>1829</td>
<td>0.110</td>
<td>0.255</td>
<td>-0.035</td>
<td>-0.170</td>
</tr>
<tr>
<td>Timber Mtn tuff</td>
<td>4846</td>
<td>1856</td>
<td>0.020</td>
<td>0.105</td>
<td>-0.030</td>
<td>-0.064</td>
</tr>
</tbody>
</table>

A simple Python program was written to plot the NMO velocity and to calculate the percentage anisotropy. Figure 3.1 shows the velocity ellipse for a model with values of $\delta^{(V)} = -0.150$, $V_P^0 = 2500$ m/s and $\beta = 0^\circ$. The percent of anisotropy for this model is 19.52%. Table 3.2 has the calculated anisotropy strength for commonly expected $\delta^{(V)}$ values. The percentage anisotropy that could be seen in practice is up to 20%. It was found that the $\beta$ angle had negligible influence on the percentage, as intuitively expected.

Table 3.2: Anisotropy percent for HTI models with horizontal reflectors.

<table>
<thead>
<tr>
<th>$\delta^{(V)}$</th>
<th>Anisotropy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.00</td>
</tr>
<tr>
<td>-0.010</td>
<td>1.02</td>
</tr>
<tr>
<td>-0.020</td>
<td>2.06</td>
</tr>
<tr>
<td>-0.050</td>
<td>5.41</td>
</tr>
<tr>
<td>-0.075</td>
<td>8.47</td>
</tr>
<tr>
<td>-0.100</td>
<td>11.80</td>
</tr>
<tr>
<td>-0.150</td>
<td>19.52</td>
</tr>
</tbody>
</table>
Chapter 3. Modelling Azimuthal Anisotropy

Figure 3.1: Velocity ellipse for a HTI medium exhibiting 19.52\% anisotropy.

The parameter $\delta^{(V)}$ has the most influence on anisotropy. It can be determined from data using additional borehole information which will give a vertical velocity and then the Thomsen parameter can be calculated using the P-wave NMO velocity (fast velocity) equation $v_{P,nmo} = v_{P0}\sqrt{1 + 2\delta^{(V)}}$.

Consider a horizontal reflector at 100 m in an anisotropic medium with $\delta^{(V)} = -0.150$ and $V_{P0} = 3000$ m/s. From Table 3.2, the percentage anisotropy in this model is 19.52\%.

Assume the data are collected along the minor velocity axis direction with an offset range of 0-200 m. The proper NMO velocity to straighten the reflector is 2509 m/s and is illustrated in Figure 3.2. Figure 3.3 shows the worst case scenario with data being corrected with a NMO velocity of 3000 m/s. This produces a time error of 0.0073 seconds in orientation of the minor axis but will produce no time error in the major axis. Figure 3.4 shows the time error shift where the minor velocity axis is oriented at 0 degrees.
Chapter 3. Modelling Azimuthal Anisotropy

Figure 3.2: Reflection times for a horizontal reflector in an anisotropic medium (blue line). The NMO corrected data used a NMO velocity of 2509 m/s (green line).

Figure 3.3: Reflection times for a horizontal reflector in an anisotropic medium (blue line). The NMO corrected data used a NMO velocity of 3000 m/s (green line).
Chapter 3. Modelling Azimuthal Anisotropy

Figure 3.4: Time error shift (\%T) for the NMO corrected data. A NMO velocity of 3000 m/s was used.

Assume that the dominant frequency in the wavelet is 100 Hz so the period of the wavelet (T) is 0.001 s. For an anisotropy of 19.52\% then a time error of 0.73T will be present in the major velocity axis for this type of target reflector. This calculation was repeated for the expected range of $\delta^{(V)}$ values and can be found in Table 3.3. Therefore, it is important to properly apply the correct NMO velocity to data. The time shifts that are introduced will reduce stack quality and can result in complete cancellation as is the case with a percentage anisotropy of 8.47\%.

Table 3.3: Time shifts as a percentage of wavelet period for common $\delta^{(V)}$ values.

<table>
<thead>
<tr>
<th>$\delta^{(V)}$</th>
<th>Anisotropy (%)</th>
<th>Time Error (s)</th>
<th>Shift (%T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.010</td>
<td>1.02</td>
<td>0.0007</td>
<td>0.07</td>
</tr>
<tr>
<td>-0.020</td>
<td>2.06</td>
<td>0.0013</td>
<td>0.13</td>
</tr>
<tr>
<td>-0.050</td>
<td>5.41</td>
<td>0.0033</td>
<td>0.33</td>
</tr>
<tr>
<td>-0.075</td>
<td>8.47</td>
<td>0.0051</td>
<td>0.51</td>
</tr>
<tr>
<td>-0.100</td>
<td>11.80</td>
<td>0.0070</td>
<td>0.70</td>
</tr>
<tr>
<td>-0.150</td>
<td>19.52</td>
<td>0.0073</td>
<td>0.73</td>
</tr>
</tbody>
</table>
3.2 Dipping Isotropic Medium

The NMO velocity for a dipping isotropic reflector described by Levin (1971) is given by

\[ v_{nmo}^2(\alpha, \phi) = \frac{v^2}{1 - \cos^2 \alpha \sin^2 \phi} \quad (3.3) \]

Figure 3.5 shows the velocity ellipse for a model with values of \( V_P = 2500 \text{ m/s} \) and a dip of \( \phi = 12^\circ \). The percent of apparent anisotropy for this model is 2.23%. The anisotropy strength for different dip angles can be found in Table 3.4. If the dip in a survey area is known, the percentage anisotropy visible in the data can be attributed to either azimuthal anisotropy or the velocity variation caused by dip or a combination of both.
FIGURE 3.5: Velocity ellipse for an isotropic dipping medium.

TABLE 3.4: Anisotropy percent for isotropic models with dipping reflectors.

<table>
<thead>
<tr>
<th>Dip $\phi$</th>
<th>Anisotropy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.015</td>
</tr>
<tr>
<td>5</td>
<td>0.382</td>
</tr>
<tr>
<td>10</td>
<td>1.54</td>
</tr>
<tr>
<td>15</td>
<td>3.53</td>
</tr>
<tr>
<td>20</td>
<td>6.42</td>
</tr>
<tr>
<td>25</td>
<td>10.34</td>
</tr>
</tbody>
</table>

It can be concluded that for data with dips less than 5 degrees, the resultant ‘apparent’ anisotropy can be ignored. However if the dip exceeds 10 degrees, the influence of the structure needs to be carefully considered. The effects of dip can be removed using the separation approach introduced by Kuehnel and Li (1996) as discussed in Chapter 2.4.
Chapter 4

Inversion of Reflection Time Data

This chapter describes an inversion algorithm which extracts anisotropic parameters from reflection time data. The algorithm will be tested using synthetic models created using the equation developed by Grechka and Tsvankin (1998). The measure for the goodness of fit that will be used for this thesis is the filter performance parameter. The reference model will be examined with errors introduced into the data and reduced data distribution.

4.1 Inversion Algorithm

Grechka and Tsvankin (1998) define the reflection time for anisotropic media as

\[ t_x^2 = t_0^2 + x^2(W_{11} \cos^2(\alpha) + 2W_{12} \sin(\alpha) \cos(\alpha) + W_{22} \sin^2(\alpha)) \]  

(4.1)

where \( t_0 \) is the zero offset time. The NMO velocity defined by

\[ \frac{1}{t_{nmo}^2(\alpha)} = W_{11} \cos^2(\alpha) + 2W_{12} \sin(\alpha) \cos(\alpha) + W_{22} \sin^2(\alpha) \]  

(4.2)

If all the traces in a common midpoint (CMP) are considered, Equation 4.1 leads to an overdetermined system \((Ax = b)\)

\[
\begin{bmatrix}
1 & x_1^2 \cos^2(\alpha_1) & 2x_1^2 \cos(\alpha_1) \sin(\alpha_1) & x_1^2 \sin^2(\alpha_1) \\
1 & x_2^2 \cos^2(\alpha_2) & 2x_2^2 \cos(\alpha_2) \sin(\alpha_2) & x_2^2 \sin^2(\alpha_2) \\
\vdots & \vdots & \vdots & \vdots \\
1 & x_{n-1}^2 \cos^2(\alpha_{n-1}) & 2x_{n-1}^2 \cos(\alpha_{n-1}) \sin(\alpha_{n-1}) & x_{n-1}^2 \sin^2(\alpha_{n-1}) \\
1 & x_n^2 \cos^2(\alpha_n) & 2x_n^2 \cos(\alpha_n) \sin(\alpha_n) & x_n^2 \sin^2(\alpha_n)
\end{bmatrix}
\begin{bmatrix}
t_0^2 \\
W_{11} \\
W_{12} \\
W_{22}
\end{bmatrix}
= 
\begin{bmatrix}
t_1^2 \\
t_2^2 \\
\vdots \\
t_{n-1}^2 \\
t_n^2
\end{bmatrix}
\]  

(4.3)
Chapter 4. Inversion of Reflection Time Data

If the CMP fold is $N$, there are $N$ equations (number of observations) and four unknowns in the system (the zero-offset time and the elements of the matrix $W$).

The matrix $W$ has three independent terms and is identified as the symmetric slowness matrix (Burnett and Fomel, 2011),

$$W = \begin{bmatrix} W_{11} & W_{12} \\ W_{12} & W_{22} \end{bmatrix} \quad (4.4)$$

The eigenvalues of the slowness matrix have the units of slowness squared and can be found using the equation

$$\lambda_{1,2} = \frac{1}{2} \left[ W_{11} + W_{22} \pm \sqrt{(W_{11} - W_{22})^2 + 4W_{12}^2} \right]$$

The major and minor velocities can be calculated from the eigenvalues.

$$v_{1,2} = \sqrt{\frac{1}{\lambda_{1,2}}} \quad (4.5)$$

The angle $\beta$ is the angle of rotation from the major velocity axis anticlockwise from the x-axis. The rotation angle can be calculated from the elements of the slowness matrix and is given by

$$\beta = \arctan \left[ \frac{W_{22} - W_{11} + \sqrt{(W_{22} - W_{11})^2 + 4W_{12}^2}}{2W_{12}} \right]$$

The NMO velocity equation can then be expressed in terms of the major and minor velocities, rotation angle and trace azimuth using

$$\frac{1}{v_{nmo}^2(\alpha)} = \frac{1}{v_1^2} \cos^2(\alpha - \beta) + \frac{1}{v_2^2} \sin^2(\alpha - \beta) \quad (4.6)$$

The inversion code will be analysed with respect to a reference model and the filter performance parameter. The filter performance parameter will be calculated based on the NMO velocity equation (Equation 4.6) and by resubstituting the inversion parameters to find the reflection time (Equation 4.3).
4.1.1 Goodness of Fit

A measure of goodness of fit is needed to determine which inversion results are most accurate. This thesis will use the filter performance parameter (FPP).

Robinson and Treitel (1980) introduced a quantity called the filter performance parameter (FPP).

\[
FPP = 1 - E
\]

\[
= 1 - \sum_t (d_t - c_t)^2
\]

\[
= 1 - \frac{\sum_t (d_t - c_t)^2}{r_{dd}(0)}
\]

where \(d\) is the recorded data, \(c\) is the recovered inverted data and \(r_{dd}\) is the autocorrelation of recorded data \(d\) at lag 0. The parameter \(E\) is a sum of squares which is always between 0 and 1 (\(0 \leq E \leq 1\)). It is termed the normalised mean square error and if it is equal to zero, there is perfect agreement with the desired and actual output. Therefore according to this new quantity, a FPP value of 1.0 is an identical match, that is \(E = 0.0\). It should be noted that the FPP value does not have a linear relationship and generally a value of at least 0.995 is needed to sufficiently match the data.
4.2 Preliminary Testing

4.2.1 Reference Model

The simple earth model used to test the inversion code included a reflector at 400 m deep and a rotation angle of 20.0 degrees. For the isotropic case, the velocity was set for 3000.0 m/s while the anisotropic case had a velocity calculated using the Equation 4.6 with a fast (major) velocity of 3200.0 m/s and a slow (minor) velocity of 2700.0 m/s. The reflection times were determined using

\[ t_x^2 = t_0^2 + \frac{x^2}{v^2} \]

where \( t_0 \) is the zero-offset time.

The method to invert for NMO velocities is described in the following subsection.

The inversion code was tested by modelling one shot point in a grid (Figure 4.1). The array was calculated using 90 geophones spaced at 10 m with 30 lines at 30 m. This was to simulate a high-resolution coal-scale operation. It should be noted the shot domain can be used in this simple case, given that the layer is horizontal and the geology is constant. In practice CMP gathers will be analysed, in an attempt to obtain anisotropy parameters in the vicinity of the nominal reflection point.

The output from the inversion is shown in Figure 4.2.

The perc parameter is the ellipticity of the curves graphed in Figure 4.3. The ellipticity is given by \( e = \frac{a-b}{a} \) where \( a \) is the major axis and \( b \) is the minor axis of the ellipse. In the isotropic case, the angle of rotation (\( \beta \)) does not hold any significance as the inversion should produce a ‘circle’. Thus, there is no velocity variation. The FPP was calculated to compare the estimated and observed values for reflection times and velocity. The estimated values for the velocity and reflection times are determined by substituting the inversion results back into Equation 4.2 and Equation 4.3 respectively. For a model with no introduced noise and full recording distribution, the inversion works perfectly (FPP=1.0) for an isotropic earth.

Figure 4.3 shows the model NMO ellipse in blue and the inversion output NMO ellipse in red. A rotational difference is clearly evident for the anisotropic case resulting in the input and output curves being shifted by 90 degrees. The velocity values returned in the inversion were switched and there is a problem with the rotation angle \( \beta \). A number of \( \beta \)
Chapter 4. Inversion of Reflection Time Data

**Figure 4.1:** Full coverage grid array to model one shot. Shot location is given by the red star and geophones are blue dots.

**Isotropic Earth**

\[
\begin{align*}
v1 &= 2999.9999999999554 \text{ m/s} \\
v2 &= 2999.9999999999764 \text{ m/s} \\
\beta &= 138.20001029201771 \text{ deg} \\
perc &= 6.9727927135925048 \times 10^{-13} \text{ %} \\
fpp_{(velocity)} &= 1.0000000000000000 \\
fpp_{(time)} &= 1.0000000000000000
\end{align*}
\]

**Anisotropic Earth**

\[
\begin{align*}
v1 &= 2699.9999999999991 \text{ m/s} \\
v2 &= 3199.99999999999864 \text{ m/s} \\
\beta &= 109.99999749552386 \text{ deg} \\
perc &= 15.624999999999670 \text{ %} \\
fpp_{(velocity)} &= 1.0000000000000000 \\
fpp_{(time)} &= 1.0000000000000000
\end{align*}
\]

**Figure 4.2:** Inversion output for reference model.
values was simulated to determine the correct orientation of the rotation angle. The results are graphed in Figure 4.4 using a polar coordinate system. An input rotation angle value of 0 degrees has the expected fast (major) velocity axis orientated at 0 degrees. The inversion results contradicts the rotation angle being measured from the fast velocity axis and is repetitively seen in the following graphs ($\beta = 30, 60, 90, 120, 150$).

Grechka and Tsvankin (1998) expressed the NMO velocity equation as

$$\frac{1}{v_{nmo}^2(\alpha)} = \lambda_1 \cos^2(\alpha - \beta) + \lambda_2 \sin^2(\alpha - \beta)$$  \hspace{1cm} (4.9)

where $\beta$ is the rotation angle and $\lambda_1, \lambda_2$ are eigenvalues of the matrix $W$. It was found in Section 4.1 that matrix $W$ was a slowness matrix, thus the eigenvalues $\lambda_1$ and $\lambda_2$ correspond to the minor and major velocity axes respectively. This explains why the inversion returns interchanged fast and slow velocities. For the major velocity axis to be situated on the x-axis (Cartesian coordinate system) for a rotation angle of 0 degrees the NMO velocity equation expressed in terms of fast and slow eigenvalues must be given by Equation 4.10.

$$\frac{1}{v_{nmo}^2(\alpha)} = \lambda_{slow} \cos^2(\alpha - \beta) + \lambda_{fast} \sin^2(\alpha - \beta)$$  \hspace{1cm} (4.10)

The inversion code calculates $\beta$ with respect to $\lambda_1$ which has been found to be the eigenvalue associated with the slow velocity axis.
FIGURE 4.4: NMO ellipse curves for reference model. A range of $\beta$ values were used to create the synthetic data to determine the meaning of the rotation angle. The input model velocity ellipse is given in blue while the output inverted ellipse is in red.
Subsequently, because the rotation angle ($\beta$) parameter is measured from the slow velocity axis, a subtraction of 90 degrees must be performed to obtain the angle relative to the fast velocity axis. The results of analysing the inversion equations can be summarised in Figure 4.5. The rotation angle relative to the fast velocity axis will be denoted $\beta_{\text{fast}}$ and will be used in following inversions as is common in literature.
4.3 Synthetic Results

Consider the effects of introducing error into the data and limiting the number of data points for the reference model. Random errors will be introduced in the time calculation stage to simulate picking errors with absolute peak values of 2 ms, 5 ms and 8 ms. The results can be found in Table 4.1. As expected the worst filter performance parameters occurred alongside the greatest random error inclusion. Generally, fewer data points gave a less accurate answer.

| TABLE 4.1: Filter performance parameter for a random data distribution in an isotropic and anisotropic earth. |
|---|---|---|
| Data Points | Random Error | ± 2 ms | ± 5 ms | ± 8 ms |
| Isotropic Earth (Velocity/Time FPP) |  |  |  |  |
| 100 | 0.99809/0.99995 | 0.99850/0.99969 | 0.9500/0.99920 |
| 300 | 0.9996/0.99995 | 0.99976/0.99966 | 0.99939/0.99914 |
| 900 | 1.0/0.99994 | 0.99999/0.99964 | 0.99998/0.99907 |
| 1500 | 1.0/0.99994 | 0.99998/0.99963 | 0.99995/0.99905 |
| 2100 | 1.0/0.99994 | 0.99999/0.99962 | 0.99997/0.99902 |
| 2700 | 1.0/0.99994 | 1.0/0.99994 | 0.99999/0.99904 |
| Anisotropic Earth (Velocity/Time FPP) |  |  |  |  |
| 100 | 0.99948/0.99995 | 0.99633/0.99967 | 0.98928/0.99915 |
| 300 | 0.99992/0.99995 | 0.99951/0.99967 | 0.99879/0.99916 |
| 900 | 1.0/0.99994 | 0.99999/0.99965 | 0.99997/0.99910 |
| 1500 | 1.0/0.99994 | 1.0/0.99960 | 1.0/0.99898 |
| 2100 | 1.0/0.99994 | 1.0/0.99960 | 1.0/0.99897 |
| 2700 | 1.0/0.99994 | 1.0/0.99962 | 0.99999/0.99902 |

The limit for data inclusion is a FPP value greater 0.995. For the reference model, a random error with a peak absolute value of 0.02 seconds resulted in a velocity FPP of 0.95 in an isotropic earth. All shot-receiver locations were included in this inversion. With a real data set, this record would be omitted from stacking.

Grechka and Tsvankin (1998) stated that the parameters for the NMO curve can be reconstructed with a minimum of three measurements from different azimuths. Figure 4.6 shows the inversion output for the reference model with minimum measurements.
Chapter 4. Inversion of Reflection Time Data

Isotropic Earth
\[ v_1 = 1394.5016969921944 \text{ m/s} \]
\[ v_2 = 3000.0000000002478 \text{ m/s} \]
\[ \beta = 89.999997495524568 \text{ deg} \]
\[ \text{perc} = 53.516610100264032 \% \]
\[ \text{fpp}_{\text{velocity}} = 0.84053737582589894 \]
\[ \text{fpp}_{\text{time}} = 1.0000000000000000 \]

Anisotropic Earth
\[ v_1 = 1365.8352876035965 \text{ m/s} \]
\[ v_2 = 3132.5484586409921 \text{ m/s} \]
\[ \beta = 91.677149699426124 \text{ deg} \]
\[ \text{perc} = 56.398590296791674 \% \]
\[ \text{fpp}_{\text{velocity}} = 0.86334088364947770 \]
\[ \text{fpp}_{\text{time}} = 1.0000000000000000 \]

Figure 4.6: Inversion output for reference model with minimum measurements.

Although the inversion gave a perfect FPP for reflection times, the velocity values were incorrect. This is reflected in the poor velocity FPP value. Thus there is a non-uniqueness problem with the inversion.

The FPP (time) value checks that the algorithm has not blown up during the inversion process. Therefore near perfect FPP (time) values ensure that the inversion algorithm has worked and produced a viable mathematical solution. The FPP (velocity) value determines the accuracy of the solution and cannot be calculated for real data. The inversion solution can be tested by applying an azimuthal NMO to see if the reflector of interest is flattened. Ensuring that there is a sufficient number of data points and that the reflection time picks are as accurate as possible, the inversion results will be more reliable even with noisy data.
4.4 Summary

The synthetic model that was analysed in this chapter has provided insight into the workings of the inversion algorithm. The algorithm can produce a solution with a minimum of three measurements. However the less data points that are inserted into the algorithm, the lower the velocity and reflection time FPP values.

In real data, only the FPP (time) value can be calculated. This measure of goodness checks the inversion algorithm has produced a mathematical solution. As discussed there is a non-uniqueness problem with the velocity solutions. The results can be validated by performing an azimuthal NMO correction on a record. If the reflector of interest is flattened, then the inversion output is feasible.
Chapter 5

Real Data Analysis

5.1 Survey Information

The 3D-3C survey was conducted in the Bowen Basin during 2009 by Velseis Pty Ltd with support of the Australian Coal Association Research Program (ACARP). The grid covered an area of 0.5 km$^2$ with 44 shot lines and 10 receiver lines. There were 392040 traces in total that were collected using 594 shots and 660 receiver locations.

5.2 Previous Studies

Hearn and Strong (2012) completed a preliminary study of azimuthal variation in PS data. The data collected in the ACARP survey was examined to determine if anisotropy was present and what extra information could be inferred.

The 3D multicomponent survey had a grid of 1200 m by 500 m with a very high fold count, approximately 500 per CCP (common conversion point) / CMP (common mid-point). The high fold allowed the data to be separated into bins based on azimuthal direction. For each CCP bin, the gathers were sorted into azimuthal ranges for automated PS-NMO parameter trials. The azimuthal variation in $\gamma$ which is the $v_P/v_S$ ratio, was determined assuming a constant $v_P$ value. The parameters which resulted in the best stack were chosen. It was expected that errors in picking reflections were significant near the fault structure which would result in smearing of the stacked volume.

Some CCP bins were averaged and any outliers were removed to improve noisy results. It was shown that the $\gamma$ averaged at 2.25 with a good correlation between mean and median values. A general NW-SE elongation in the $\gamma$ velocity ellipse was present near the central
fault. The $\gamma$ anisotropy had an order of 5-10%. The region has reverse faulting, thus the stress orientation should be perpendicular to strike. The interpreted high $\gamma$ direction was approximately in the strike-direction of faulting. The results suggested that the method allowed for the detection of anisotropy in data however, it was inconclusive whether the anisotropy was related to $v_P$, $v_S$ or both.

Strong (2016) re-examined the results including the implications of the constant $v_P$ assumption made in the earlier study. If $v_P$ was allowed to vary, then a 7% $\gamma$ variation would be consistent with a range of possible combinations. It can be seen from Table 5.1 that the observations in the data are non-unique. In addition, the azimuthal-binning methodology used in the original work was subject to error. This was mainly because the fold in each azimuthal bin can be very small. For these reasons, the earlier work can be taken as an indication that azimuthal anisotropy is likely but the exact dependence on $v_P$ and $v_S$ required further investigation.

<table>
<thead>
<tr>
<th>$v_P$ anisotropy (%)</th>
<th>Orientation</th>
<th>$v_S$ anisotropy (%)</th>
<th>Orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>NE-SW</td>
<td>7</td>
<td>NE-SW</td>
</tr>
<tr>
<td>5</td>
<td>NE-SW</td>
<td>5</td>
<td>NE-SW</td>
</tr>
<tr>
<td>15</td>
<td>NE-SW</td>
<td>0</td>
<td>NE-SW</td>
</tr>
<tr>
<td>20</td>
<td>NE-SW</td>
<td>3</td>
<td>NW-SE</td>
</tr>
</tbody>
</table>

### Table 5.1: Possible combinations for a $\gamma$ variation of 7%. Adapted from Strong (2016).

#### 5.3 Data Processing

The ACARP data went through minimal pre-processing which was completed using PROMAX and included applying geometries, sorting into CMP domain and residual and refraction statics. Two main areas were chosen for analysis in this thesis. Area 1 is 300 m by 100 m and is located approximately in the middle off the survey as pictured in Figure 5.1. The second area covers the length of a receiver line and will be referred to as Line 1.

#### 5.3.1 Travel Time Picking

This study examines P-wave anisotropy by inverting P-wave reflection times. The picking of the reflection horizon was completed using the Seismic Unix processing module suxpicker. The parameters needed to calculate the offset, shot and common mid-point
Figure 5.1: The CMP locations for the survey are indicated by the black dots. Area 1 covers 300m by 100m (red cross) and Line 1 is 1 km in length (green cross).

```bash
cat acarp_vert_sgy_cdpsort.xdr | suwind key=cdp min=$1 max=$1 | sugain qbal=1 | sushw key=tracl a=1 b=1 |
sugethw key=tracl,sx,sy,gx,gy output=geom >$1.geom
```

Figure 5.2: Seismic Unix commands used in the picking process.

(CMP) locations were $sx$, $sy$, $gx$ and $gy$. The values of $tracl$ are used to find the appropriate pick time for the correct geometry data. Figure 5.2 shows the commands used to download the geometries and the picking parameters used to determine the horizon. An example gather (CDP1231) is graphed in Figure 5.3. The reflector of interest in the ACARP data is approximately at 0.1 seconds on near offsets.

The CDP1231 reflector was picked using two methods - picking at peak amplitudes or at the zero crossing. The picks are illustrated in Figure 5.4. Although the inversion (Figure 5.5) returns parameters with similar results, the picking method that will be used will be picking of the peaks. For Vibroseis data the wavelet is not minimum phase like explosive sources. The autocorrelation has the form of a Klauder wavelet which is zero phase. The data were obtained with a Vibroseis source, therefore a more accurate inversion should occur when picking a horizon at the peak amplitude. The results for the zero crossing picks had an average bulk shift applied to determine if zero-picking could be
FIGURE 5.3: The reflector of interest for CDP1231 occurs at approximately 0.1 seconds on near offsets.

completed as it can be easier and more efficient. However when examining the peak amplitude time length to determine the bulk shift, it was found that the time length for different traces was not consistent. This produced a small increase in the filter performance parameter (Figure 5.6) but not enough to change the picking method. For these data, picking at the peak amplitude appears to be a more accurate method and will be used for the analysis of the remaining data.
Figure 5.4: CDP1231 picks using suxpicker. Top: Peak amplitude picks. Bottom: Zero crossing picks.
CDP1231 - Picking peak amplitudes
\begin{align*}
v1 &= 2900.16260 \text{ m/s} \\
v2 &= 2871.91040 \text{ m/s} \\
\beta &= 90.6930695 \text{ deg} \\
perc &= 0.974159062 \% \\
fpp &= 0.999318838
\end{align*}

CDP1231 - Picking at zero crossing
\begin{align*}
v1 &= 2902.95825 \text{ m/s} \\
v2 &= 2872.32886 \text{ m/s} \\
\beta &= 85.0668793 \text{ deg} \\
perc &= 1.05510974 \% \\
fpp &= 0.999114931
\end{align*}

**Figure 5.5:** Inversion results for picking methods.

CDP1231 - Picking at zero crossing (bulk shift)
\begin{align*}
v1 &= 2864.05884 \text{ m/s} \\
v2 &= 2834.32104 \text{ m/s} \\
\beta &= 85.3367615 \text{ deg} \\
perc &= 1.03830945 \% \\
fpp &= 0.999154568
\end{align*}

**Figure 5.6:** Inversion results for picking at zero crossing (bulk shift).

### 5.3.2 Inversion

The inversion algorithm used on this data set was introduced in Chapter 4.1. The geometry information for each trace was written to a data file. The *trac* number with its corresponding shot and receiver coordinates were matched to the picks for each CDP location. These data became the input for the inversion code. The inversion output the CDP number, major and minor velocity, slow rotation angle, percent anisotropy and filter performance parameter.

Figure 5.7 shows the goodness of fit for the data. The filter performance parameter is greater than 0.9970 for each CDP location. From Chapter 4.1.1, this FPP value shows that the results collected from the inversion algorithm are mathematically valid. None of the data needs to be omitted before analysis is undertaken.
5.4 Data Analysis and Interpretation

The inversion algorithm returned the fast and slow velocity, slow rotation angle and percent anisotropy. This information will be used to construct the effective NMO ellipses. The ellipse plots required for analysis were created by plotting straight lines at the correct fast angle orientation and scaling these by the percent of anisotropy. The line is in the direction of the fast (major) velocity axis and a smaller line shows less anisotropy. A plot of the ellipses for Area 1 and Line 1 is illustrated in Figure 5.8.

Each area will be examined individually and plotted using the most appropriate visualisation for the data.

5.4.1 Area 1

The effective NMO ellipses for Area 1 is in Figure 5.9. It can be seen that for most of the data, the fast direction follows a NW-SE orientation. In theory, this points in the direction of maximum horizontal stress. This trend is more prominent in the northern part where there is higher anisotropy. The northern part of Area 1 included the record with the highest anisotropy percentage of 9.3% which is significant. The southern part of the survey area is more irregular. This is due to the greater noise seen in the data which could cause the
Figure 5.8: Azimuthal plot of all CDP locations. Area 1 had a highest percentage anisotropy of 9.3%. Line 1 had a highest percentage anisotropy of 9.6%. 
variation in axis orientation. The average percentage anisotropy in this area was 3.3%.

The survey location has a dip of approximately 5 degrees. As discussed in Chapter 3, the expected anisotropy percent for a dip of 5 degrees is less than 0.5%. In addition, the dip direction in the survey area is SW-NE which is not consistent with the observed anisotropy. Therefore, the variation that is seen in the data is not likely to be caused by the presence of dip and the anisotropy found appears to indicate true azimuthal anisotropy.

As discussed above, previous work (Hearn and Strong, 2012) also suggested an anisotropy trend (in $v_P/v_S$) elongated in a NW-SE orientation. That analysis assumed a constant $v_P$ ad so the true significance in terms of $v_P$ and/or $v_S$ anisotropy was ambiguous. Nevertheless it is interesting that the $v_P$ trend seen in this study is also NW-SE. It is thought that the inversion algorithm used here should be more reliable than the azimuthal binning approach used in the earlier study. The inversion approach needs to be extended to the full survey area. A more extensive study will be required to confirm the orientation trend of the whole survey.
Figure 5.9: Azimuthal plot of Area 1. The blue line in the legend represents a percentage anisotropy of 14%.
5.4.2 Line 1

To examine the inverted data along a receiver line, an azimuthal NMO will be performed on each CMP location using Seismic Unix. This output will then be stacked to visualise the reflector along Line 1.

A preliminary assessment of the inversion result was performed by comparing a standard NMO correction and azimuthal NMO correction on a high anisotropy CMP location. The azimuthal NMO correction was completed in Seismic Unix by taking the data for a CMP location and acting like it was a complete record, i.e. each trace was a new CMP. The azimuth for each new CMP was calculated using the geometry obtained in the header information and the velocity correction for the new CMP was determined using the azimuthal velocity equation (Equation 5.1) where the slow and fast velocity and the rotation angle is the inversion output.

\[
\frac{1}{v_{\text{nmo}}^2} = \frac{1}{v_{\text{slow}}} \cos^2(\alpha - \beta) + \frac{1}{v_{\text{fast}}} \sin^2(\alpha - \beta) \quad (5.1)
\]

The calculated velocity for each new CMP will be the input for the routine sunmo. The CMP locations that was chosen to be examined were CDP1310 and CDP1339. The anisotropy percentage for these gathers are 4.7% and 8.6% respectively. The NMO corrected gather for CDP1310 can be found in Figure 5.10 and Figure 5.11. Figure 5.12 and Figure 5.13 illustrate the correction for CDP1339. When digital versions of the images are toggled, it appears that the azimuthal correction provides improved alignment.
After applying an azimuthal correction to each CMP gather in Line 1 a final stack was produced. The highest percentage of anisotropy along the receiver line was 9.60% but averaged at 3.61%. Similarly to Chapter 3.1, a maximum time error of 0.23T is expected to be present in the data for a standard NMO correction. It is likely not this large as the NMO velocity which gave the straightest reflector was chosen. It was assumed that the dominant frequency of the Vibroseis source is 100 Hz. A time error of 0.23T should result in partial cancellation of the wavelet. Thus by accounting for the velocity variation a better stack image should be produced.

The azimuthal NMO stack is illustrated in Figure 5.15. The standard NMO correction stack (Figure 5.14) for Line 1 was also calculated to highlight any similarity or differences in the final product. The stacks have a clear reflector along the extent of Line 1 around 0.1 seconds. There is a fault in the reflector around CDP1319. The stacks show minimal difference and are hard to discern from one another.

Based on examination of CMP gathers, the offset was limited to a range of 100-200 metres, since this is the zone of strongest reflections. The stacked images for the standard NMO correction (Figure 5.16) and the azimuthal NMO correction (Figure 5.17) again showed minimal difference. The frequency content of the standard NMO and azimuthal NMO offset-limited stack records are illustrated in Figure 5.18 and Figure 5.19 respectively. There is minimal difference in the frequency content.

Clearly the improvements seen in the gathers are not carried through into stacks. It appears that the improvements in the NMO are being counteracted by other processing deficiencies (e.g. statics, NMO stretch).
Figure 5.10: Standard NMO corrected data for CDP1310.
Figure 5.11: Azimuthal NMO corrected data for CDP1310.
Figure 5.12: Standard NMO corrected data for CDP1339.
Figure 5.13: Azimuthal NMO corrected data for CDP1339.
FIGURE 5.14: Standard NMO corrected data stack.
Figure 5.15: Azimuthal NMO corrected data stack.
FIGURE 5.16: Standard NMO corrected data stack with limited offset.
Figure 5.17: Azimuthal NMO corrected data stack with limited offset.
FIGURE 5.18: Frequency content of standard NMO corrected stack for limited offset.
FIGURE 5.19: Frequency content of azimuthal NMO corrected data stack for limited offset.
5.5 Summary

The inversion algorithm was tested on real data from a 3D-3C survey in the Bowen Basin. Area 1 was located in the middle of the survey and had a clear trend evident in the northern section with a NW-SE orientation. The percentage anisotropy in Area 1 averaged at 3.3% with the largest anisotropy around 9%. There was dip present in the survey area of approximately 5 degrees. However it was found that the anisotropy was not likely to be caused by dip effects. Therefore, it appears that true azimuthal anisotropy is present. It was concluded that the methodology employed in this thesis was able to determine the orientation of the NMO ellipse and the percentage anisotropy.

Line 1 was examined to visualise the reflector along the survey and to find if an improvement in stack quality is evident when applying an azimuthal NMO correction. The percentage anisotropy along the receiver line averaged at 3.6%. The individual CMP gathers that were corrected with an azimuthal NMO showed an improvement in straightening the reflector over the standard NMO correction. It was expected from seeing the improvements in the gathers that accounting for azimuthal anisotropy in the data would result in a better stacked image. The improved stack quality was not evident for either full and offset-limited stacks. The frequency of the offset-limited stacks was examined to visualise any increase in frequency content. Again there was minimal difference in the stacks. It was believed that the quality expected could have been counteracted by several factors including statics, NMO stretch and complicated ray paths.
Chapter 6

Converted-wave Analysis

S waves travel differently in a medium to P waves which could provide additional insight on the subsurface properties. Converted-wave data require multicomponent seismic recording equipment, and analysis is more complex.

A converted-wave data set for the ACARP survey area (Chapter 5) was available for analysis. A small number of CCP gathers have been examined to provide a preliminary indication of the applicability of the inversion algorithm to converted-wave data. The minimal pre-processing which was completed using PROMAX included applying geometries, coordinate rotation in the radial direction, P-wave source and S-wave receiver statics, preliminary velocity analysis for binning and horizon based CCP binning. The CCP gathers picked for inversion were chosen to overlap the P-wave data in the northern part of Area 1. Figure 6.1 shows Area 2 (black) in relation to Area 1 (red).
Figure 6.1: The CDP locations for the survey are indicated by the black dots. Area 1 (red cross) and Area 2 (black cross).

6.1 Theory Overview

A converted-wave reflection usually has a P-wave source that converts to a S-wave upon hitting a reflector. The path the wave travels is asymmetric and the reflection point is known as the common conversion point (CCP) which is located closer to the receiver.

Thomsen (1999) described the reflection time for a converted-wave in a homogeneous single layered earth as

$$t_{xc}^2 = t_{c0}^2 + \frac{x_o^2}{v_c^2}$$  \hspace{1cm} (6.1)

where $t_{c0} = t_{p0} + t_{s0}$ is the zero-offset travel time, $x_o$ is the offset and $v_c^2 = \frac{v_p^2}{1+v_p/v_s} + \frac{v_s^2}{1+v_s/v_p}$ is the velocity.

The CCP can be calculated to the first approximation (Tessmer and Behle, 1988) using

$$x_p = \frac{x_o}{1 + \frac{2}{v_s}}$$  \hspace{1cm} (6.2)

where $x_p$ is the horizontal reflection point and $x_o$ is the source to receiver offset.
Chapter 6. Converted-wave Analysis

When an incident S-wave travels through an anisotropic media, it splits into two polarised waves (parallel and perpendicular) with two separate velocities. This is known as shear-wave splitting or seismic birefringence.

Grechka et al. (1999) proved that a shifted converted-wave NMO velocity has the same elliptical form as a P-wave NMO velocity. The original coordinate system for the converted-waves is shifted to a traveltime minimum. The NMO velocity for this new CMP location is given by

\[
[V^*(PS)]_{nmo}^{-2} = W_{11}^{(PS)} \cos^2(\alpha) + 2W_{12}^{(PS)} \sin(\alpha) \cos(\alpha) + W_{22}^{(PS)} \sin^2(\alpha)
\]

\[
= \lambda_1 \cos^2(\alpha - \beta) + \lambda_2 \sin^2(\alpha - \beta)
\]

where \( \beta \) is the rotation angle, \( \alpha \) is the source-receiver azimuth and \( \lambda_1, \lambda_2 \) are eigenvalues of the matrix \( W \).

The converted-wave data in this thesis will used the same inversion algorithm as the P-wave data. Based on Equation 6.3 and 6.4, the P-wave algorithm should successfully indicate the overall azimuthal parameters (strength, direction). The detailed significance in terms of individual \( v_P \) and \( v_S \) anisotropy will require further research.

6.2 Data Processing

The picking of the reflection horizon for the converted-wave data was completed identically to the P-wave data. The geometry data (shot and receiver location) had to be calculated in a different manner because the SU file only had the receiver and shot station and line numbers in the headers. The correct geometries for each time picked had to be determined from files containing the receiver and shot locations. Figure 6.2 shows the commands used to download the geometries and the picking parameters. An example gather (CDP504) is graphed in Figure 6.3. The reflector is approximately at 0.15 seconds on near offsets. To make picking easier, the polarity of the data was reversed.

Twelve CCP gathers were picked and ran through the same inversion algorithm as the P-wave data. The average anisotropy seen in the Area 2 was 4.50%. An azimuthal NMO correction (Figure 6.5) was calculated and compared to a standard NMO correction (Figure 6.4) for the CCP gather with the highest anisotropy. CDP658 had a percentage
Chapter 6. Converted-wave Analysis

FIGURE 6.2: Seismic Unix commands used in the picking process.

FIGURE 6.3: The reflector of interest for CDP504 occurs at approximately 0.15 seconds on near offsets.
anisotropy of 8.30%. It is difficult to determine between the two NMO methods as the converted-wave data have considerable noise.

Figure 6.4: Standard NMO corrected data for CDP658.

Figure 6.5: Azimuthal NMO corrected data for CDP658.
Figure 6.6 shows the goodness of fit for the data in Area 2. The filter performance parameter shows that the inversion worked correctly and has produced a solution. No data points will be omitted.

The effective NMO ellipses for Area 1 and Area 2 are in Figure 6.7. The azimuthal orientation trend of NW-SE for the converted-wave data is consistent with the P-wave data.

### 6.3 Summary

A preliminary analysis of twelve CCP gathers was completed using the same inversion algorithm as the P-wave data. The algorithm was able to produce a mathematical solution for each gather, therefore no results were omitted. Area 2 had an average percentage anisotropy of 4.5% and the individual gathers had velocity variations of a similar order to the P-wave data for northern part of Area 1. The elongation of the velocity ellipse was oriented NW-SE which was consistent with Area 1.
Figure 6.7: Azimuthal plot for Area 1 (P-wave data) and Area 2 (converted-wave data). The blue and red lines in the legend represent a percentage anisotropy of 14%. 
Chapter 7

Conclusion

7.1 Summary

Azimuthal anisotropy is the azimuthal dependence of seismic velocities in a medium. The normal moveout velocity shows elliptical variation and if not accounted for during seismic processing, it is expected to cause significant misalignment and interference in stacking. Azimuthal anisotropy can be caused by stress or fractures, which in theory, aligns the major velocity axis to the maximum horizontal stress orientation. Thus by analysing the NMO ellipse, anisotropic information relating to fracturing can be obtained. A coal-scale 3D data set was used to determine how significant the effects of azimuthal anisotropy are and what information can be extracted.

The effects of azimuthal anisotropy are not limited to a true anisotropic medium and can be caused by a dipping reflector in an isotropic medium. The percentage anisotropy that is expected for a dipping reflector needs to be considered if the dip in a survey area exceeds 10 degrees. Otherwise the percentage anisotropy is less than 1.5%.

The time shift of a wavelet was calculated for several anisotropy strengths. This determined that if the percentage anisotropy were around 8.5%, total cancellation would occur, significantly reducing stack quality for a Vibroseis source. The inversion algorithm described by Grechka and Tsvankin (1998) was examined in detail. The inversion algorithm determines the values of the zero-offset time and the elements of a slowness matrix. These elements can be used to calculate a rotation angle and the major and minor velocities of the ellipse. The meaning of the rotation angle was defined in different ways in the literature. It was found that the eigenvalues of the slowness matrix determined which axis the rotation angle was defined from. If the first eigenvalue calculated the minor velocity then the rotation angle was measured from the minor axis. This allowed the results
of the real data set to be confidently stated.

The P-wave data used in the thesis were from a coal-scale 3D seismic reflection set. Two areas were chosen for analysis, Area 1 and Line 1. Area 1 was located in the middle of the survey and consisted of 109 records. Line 1 extended the length of one receiver line in the middle of the survey. The NMO ellipses were calculated for Area 1 and showed a preferred orientation of NW-SE. The dip in the area was known to be approximately 5 degrees and the percentage anisotropy was at least 6% in the northern part. Thus it was determined that the anisotropy seen in Area 1 appears to indicate true azimuthal anisotropy. The analysis of Line 1 included applying a standard and azimuthal NMO correction followed by stacking. The stacks with limited offset had their frequency content examined. Azimuthal NMO appears to improve event alignment on CMP gathers. However the improvement is not seen in stacks. The expected improvements could have been counteracted by statics and NMO stretch.

A preliminary analysis of converted-wave data from the same area suggests anisotropy which appears consistent with the P-wave results. The detailed significance in terms of $v_p$ and/or $v_S$ anisotropy requires further research.

### 7.2 Recommendations for Future Study

For the P-wave data set, it is recommended that the full survey should be analysed using the inversion algorithm. A comparison for basic and azimuthal NMO corrections should be completed for the remainder of the receiver lines. Several other coal-scale data sets should be analysed as well to determine whether a significant improvement in stacking quality is evident for shallow reflectors.

For the converted-wave data set, it is recommended that a full azimuthal analysis be completed over the whole survey. Methods for extracting the S-wave portion of the data from the known P-wave analysis should also be examined. This could give valuable information on the subsurface that would be beneficial to exploration.
Appendix A

Inversion Code

LISTING A.1: Array set up.

SUBROUTINE array ( line, geophone, dx, dy, shotx, shoty, xoffset, & azimuth 
! Shakira Heffner 11/05/15
! Subroutine sets up array centered on a cartesian coordinate system.
! Returns distance from shot to receiver and azimuth angle (radians).
! Inputs: line − number of receiver lines
! geophone − number of geophones per line
! dx, dy − distance between geophones (dx) and lines (dy)
! shotx, shoty − coordinates of shot point
! Outputs: xoffset − distance from the shot to receiver (array)
! azimuth − azimuth angle relative to East (array)
IMPLICIT NONE
INTEGER, INTENT(IN) :: line, geophone
INTEGER :: i, j, k ! counters, k − numbering of geophones
REAL*8, INTENT(IN) :: shotx, shoty, dx, dy
REAL*8, DIMENSION(line*geophone) ! counters for geophones
REAL*8, INTENT(OUT) :: xoffset, azimuth
REAL*8 :: xloc, yloc ! coordinates for geophone
REAL*8 :: disx, disy ! distance components for shot−receiver
REAL*8 :: angle ! azimuth angle dummy variable
REAL, PARAMETER :: PI = 4.0*ATAN(1.0)
DO i=1, line, 1
  DO j=1, geophone, 1
    xloc=(geophone−2)*dx/2.0+dx*(j−1)
    yloc=(line−2)*dy/2.0−dy*(i−1)
    disx=xloc−shotx
    disy=yloc−shoty
    k=j+(i−1)*geophone
    xoffset(k)=SQRT(disx*disx+disy*disy)
    angle=ATAN2(disy, disx)
    azimuth(k)=angle
  END DO
END DO
END SUBROUTINE array

LISTING A.2: NMO velocity inversion using SVD.

SUBROUTINE inversion ( xoffset, azimuth, rtime, total, v1, v2, & beta, perc, fit )
! Shakira Heffner 30/08/15
! Subroutine inverts data using SVD to determine
! velocity (major and minor), direction of fracture (beta) and
Appendix A. Inversion Code

percentage anisotrophy (perc).

Inputs:
- xoffset - distance from the shot to receiver (array)
- azimuth - azimuth angle relative to East (array)
- rtime - reflection times (array)

Outputs:
- v1, v2 - velocity major and minor axis
- beta - direction of fractures (radian)
- perc - percentage anisotrophy

IMPLICIT NONE

INTEGER :: i, j, k ! counters
INTEGER, INTENT(IN) :: total
REAL*8, INTENT(OUT) :: v1, v2, beta, perc, fit
REAL*8, DIMENSION(total) :: xoffset, azimuth, rtime
REAL*8, DIMENSION(total) :: right
REAL*8, DIMENSION(4, total) :: matrix, matrixcopy
REAL*8, DIMENSION(4) :: righthat
REAL*8, DIMENSION(4) :: ans
REAL*8 :: btop, lambda1, lambda2, wmax, thresh
REAL*8, PARAMETER :: PI = 4.0*ATAN(1.0)
REAL*8, PARAMETER :: TOL = 1E-7

DO i=1, total, 1
  matrix(i,1)=DBLE(1.0)
  matrix(i,2)=xoffset(i)**2*COS(azimuth(i))**2
  matrix(i,3)=2.0*xoffset(i)**2*COS(azimuth(i))*SIN(azimuth(i))
  matrix(i,4)=xoffset(i)**2*SIN(azimuth(i))**2
  right(i)=rtime(i)**2
END DO

DO i=1, total, 1
  matrixcopy(i,1)=matrix(i,1)
  matrixcopy(i,2)=matrix(i,2)
  matrixcopy(i,3)=matrix(i,3)
  matrixcopy(i,4)=matrix(i,4)
END DO

CALL svdcmp(matrix, total, 4, total, 4, w, v)

wmax=0.0
DO j=1, 4
  IF (w(j).GT.wmax) wmax=w(j)
ENDDO
WRITE(*,*) w
thresh=TOL*wmax
DO j=1, 4
  IF (w(j).LT.thresh) w(j)=0.0
ENDDO
WRITE(*,*) w

CALL svbksb(matrix, w, v, total, 4, total, 4, right, ans)

Calculates FPP

righthat=MAVMUL(matrixcopy, ans)

CALL filterpp(total, right, righthat, fit)

lambda1=0.5*(ans(2)+ans(4)+SQRT((ans(2)-ans(4))**2+&4.0*ans(3)**2))
lambda2=0.5*(ans(2)+ans(4)-SQRT((ans(2)-ans(4))**2+&4.0*ans(3)**2))
v1=(1.0/lambda2)**(1.0/2)
v2=(1.0/lambda1)**(1.0/2)
btop=ans(4)-ans(2)+SQRT((ans(4)-ans(2))**2+4.0*ans(3)**2)
beta=ATAN2(btop, (2.0+ans(3)))
Appendix A. Inversion Code


SUBROUTINE filterpp (ld, d, c, fpp)
! Program calculates the filter performance parameter between two signals.
IMPLICIT NONE
INTEGER, INTENT(IN) :: ld ! length of signal
INTEGER :: i, j ! counters
REAL*8, INTENT(OUT) :: fpp ! FPP
REAL*8, INTENT(IN) :: d(+), c(+), ! arrays for signals
REAL*8 :: diff, sum, rdd0 ! difference, sum and cross-correlation

sum=0.0
DO i=1, ld, 1
  diff=d(i)−c(i)
  sum=sum+diff∗diff
END DO
rdd0=0.0
DO j=1, ld, 1
  rdd0=rdd0+d(j)∗d(j)
END DO
fpp=1.0−sum/rdd0
END SUBROUTINE filterpp

LISTING A.4: P-wave isotropic reflection times (for modelling).

SUBROUTINE isotimes (rdepth, vearth, xoffset, total, time)
! Shakira Heffner 22/08/15
! Subroutine calculates the reflection times for a homogeneous, isotropic
! Earth (used for modelling).
! Inputs:  rdepth – reflector depth
!         vearth – velocity of Earth
!         xoffset – offsets (array)
!         total – number of data points
! Outputs: time – reflection times (array)
IMPLICIT NONE
INTEGER :: i ! counters
INTEGER, INTENT(IN) :: total
REAL*8, INTENT(IN) :: rdepth, vearth
REAL*8, DIMENSION(total), INTENT(IN) :: xoffset
REAL*8, DIMENSION(total), INTENT(OUT) :: time
REAL*8 :: to ! travel time at zero offset

to=(2.0∗rdepth)/vearth
DO i=1, total, 1
  time(i)=SQRT(to**2+xoffset(i)**2/vearth**2)
END DO
END SUBROUTINE isotimes

LISTING A.5: P-wave anisotropic reflection times (for modelling).

SUBROUTINE anisotimes (rdepth, vearth, vnmo, xoffset, total, time)
! Shakira Heffner 22/08/15
! Subroutine calculates the reflection times for an anisotropic
! Earth (used for modelling).
! Inputs:  rdepth – reflector depth

perc=(v1−v2)/v1∗100.0
END SUBROUTINE inversion
Appendix A. Inversion Code

IMPLICIT NONE

INTEGER :: i ! counters
INTEGER, INTENT(IN) :: total
REAL (*8, INTENT(IN)) :: rdepth, vearth
REAL (*8, DIMENSION(total), INTENT(IN)) :: xoffset, vnmo
REAL (*8, DIMENSION(total), INTENT(OUT)) :: time
REAL *8 :: to ! travel time at zero offset

to = (2.0*rdepth) / vearth

DO i = 1, total, 1
    time(i) = SQRT(to**2 + xoffset(i)**2 / vnmo(i)**2)
END DO

END SUBROUTINE anisotimes

LISTING A.6: NMO velocity (for modelling).

SUBROUTINE velnmo ( vmajor, vminor, azimuth, beta, total, vnmo)

! Shakira Heffner 22/08/15
! Subroutine calculates the velocity for an anisotropic Earth.
! 1/vnmo**2 = 1/v1**2*cos**2(alpha-beta) + 1/v2**2*sin**2(alpha-beta)
! Inputs: vmajor/vminor — velocity axis
!         azimuth — azimuth (radians) relative to East (array)
!         beta — direction of the fractures (radians)
!         total — number of data points
! Outputs: vnmo — nmo velocity (array)

IMPLICIT NONE

INTEGER :: i ! counter
INTEGER, INTENT(IN) :: total
REAL (*8, INTENT(IN)) :: vmajor, vminor, beta
REAL (*8, DIMENSION(total), INTENT(IN)) :: azimuth
REAL (*8, DIMENSION(total), INTENT(OUT)) :: vnmo
REAL *8 :: cosdum, sindum ! velocity dummy variables

DO i = 1, total, 1
    cosdum = COS(azimuth(i) - beta) / vmajor
    sindum = SIN(azimuth(i) - beta) / vminor
    vnmo(i) = SQRT(1.0 / (cosdum**2 + sindum**2))
END DO

END SUBROUTINE velnmo
References


