Question 1

(a) Two masses \( m_1 \) and \( m_2 \) are separated by a distance \( r \). Write down Newton’s Law of Universal Gravitation, defining any extra parameters. (1)

(b) Derive an expression for the gravitational acceleration experienced by the body of mass \( m_2 \). (2)

(c) The term gravitational potential refers to the gravitational potential energy of a 1kg mass. Derive an expression for the gravitational potential at a distance \( r \) from a mass \( m_1 \). (4)

(d) Assume that to a first approximation the earth acts as if all its mass was at the centre. Calculate the average gravitational potential at the surface of the earth, and at a point 1m above the surface. Include enough significant figures to show the difference in the two potentials. Note the units. (3)

(e) Use the results of (d) to estimate the gravitational acceleration (magnitude and direction) at the surface of the earth. (2)

(f) A gravity survey is being carried out in Mongolia (Latitude = 47° N). Gravity readings are being adjusted to allow for the effects of drift, latitude and elevation. Consider the corrections being made at Stations A and C, relative to a base station B:

<table>
<thead>
<tr>
<th>Station</th>
<th>Reading</th>
<th>Time</th>
<th>Distance N/S of B</th>
<th>Elevation rel.B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>39305 gu</td>
<td>0800</td>
<td>10 km S</td>
<td>-20m</td>
</tr>
<tr>
<td>C</td>
<td>39242 gu</td>
<td>1600</td>
<td>5 km N</td>
<td>15m</td>
</tr>
</tbody>
</table>

At the base station B, the readings at times 0800 and 1600 were 39205 gu and 39202 gu respectively. For the Bouguer correction, assume a typical granitic density of 2650 kg m\(^{-3}\).

Deduce the Bouguer corrected gravity values at A and C relative to the base station B. (8)
Question 2

The gravitational anomaly due to a buried sphere has a vertical component given by

\[ g_Z = \frac{4\pi G \sigma R^3 Z}{3(x^2 + Z^2)^{3/2}} \]

where the symbols have their usual meanings.

(a) Sketch the general shape of the anomaly due to a buried sphere. Annotate the maximum anomaly, in terms of the parameters in the equation above. On your sketch, also clearly define the half-width \( x_{1/2} \).

(b) Model A is a sphere whose parameters are \( R=50\text{m}, Z=100\text{m}, \sigma = 1000 \text{ kg m}^{-3} \). Model B produces an identical gravity anomaly, and has a value of \( \sigma = 500 \text{ kg m}^{-3} \). What are the other model parameters (\( R \) and \( Z \)) for Model B?

(c) For the case of a buried sphere, derive a relationship between the half-width \( x_{1/2} \) and the depth to the centre of the sphere \( Z \).

(d) Drilling reveals that a prospective ore body has a density of 3700 kg m\(^{-3}\), while the country rock has a density of 2700 kg m\(^{-3}\). A gravity survey over the body results in the profile shown in Figure 1. Assuming that the body is approximately spherical in shape, estimate the total mass of the ore.

![Figure 1: Vertical gravity anomaly over an ore body](image-url)
Question 2 (continued)

(e) The total-field magnetic anomaly due to a horizontal cylinder, striking EW, and situated on the equator is

\[ T(x) = \frac{B_M R^2 (x^2 - Z^2)}{2(x^2 + Z^2)^2} \]

where the symbols have their usual meanings. The following is a section of a Python program designed to compute and plot the anomaly as a function of horizontal coordinate (x).

```
    R=45.0    #radius of cylinder (m)
    Z=120.0   #depth to centre of cylinder (m)
    Bh=50000.0 #\mu_0 H (nT)
    k=0.001   #SI
    xstart=-300.0
    xincrement=1.0
    N=600
    x=zeros(N,float)  # array for x coordinates
    T=zeros(N,float)  # array for total field
    for i in range (0,N):
        x[i]=xstart+i*xincrement
```

Insert the line(s) of code to construct the array \( T \) holding the total-field magnetic anomaly. Indicate clearly where your new code is positioned relative to the supplied code. (4)
Question 3

(a) The magnetic B field and H field are related via the permeability of the medium:

\[ B = \mu H. \]

By introducing the concepts of vacuum permeability, relative permeability and susceptibility, show that the observed B field is made up of a component \( B_H \) which would occur in a vacuum, and an induced component \( B_M \) resulting from the susceptibility of the material. That is

\[ B = B_H + B_M \]

where \( B_H = \mu_0 H \) and \( B_M = \mu_0 M \)

In your answer, clearly define the terms \( \mu_0, \mu_r, \kappa \) and \( M \)

(b) The SI unit of \( B, B_H \) and \( B_M \) is the Tesla (N A\(^{-1}\) m\(^{-1}\)). The unit of \( \mu_0 \) is N A\(^{-2}\). Show that the unit of \( H \) (and \( M \)) is A m\(^{-1}\).

(c) Consider an area where the country rocks have negligible magnetic susceptibility \( \kappa \). If the magnitude of the observed B field is 53500 nT, find

(i) magnitude of \( B_H \)
(ii) magnitude of \( H \)
(iii) magnitude of \( B_M \)
(iv) magnitude of \( M \)

(d) An igneous intrusion in the same area has a susceptibility of 0.1 (SI). In this location find

(i) magnitude of \( B_H \)
(ii) magnitude of \( H \)
(iii) magnitude of \( B_M \)
(iv) magnitude of \( M \)

(e) At a particular location in the vicinity of the intrusion, the \( B_M \) vector is at an angle of 150° to the \( B_H \) vector, such that partial cancellation occurs. Sketch the \( B_H \) and \( B_M \) vectors. Find the magnitude of the measured B field.

(f) A high-susceptibility dyke-like body strikes EW, at a location where the inclination is -45°. Sketch the earth’s field and the induced field, and hence deduce the general shape of the observed magnetic anomaly.
Question 4

I want to drive a tractor from Point A to Point C across two paddocks, in the shortest possible time. Paddock 1 is plowed and I can travel at speed $v_1$. Paddock 2 is firmer and I can travel faster, at speed $v_2$. The geometry of the situation is summarised in Figure 2. I need to determine the point B (at a distance $x$ to the left of A) which will minimise my time.

Figure 2: Path from A to C (in red) across two paddocks.

(a) Using the symbols marked on Figure 2, prove that the time taken to travel from A to C via B is

$$T = \frac{\sqrt{x^2 + h_1^2}}{v_1} + \frac{\sqrt{(L-x)^2 + h_2^2}}{v_2}$$

(2)

(b) Show that the time from A to C is minimised when $x$ satisfies the expression

$$\frac{1}{v_1} \frac{x}{\sqrt{x^2 + h_1^2}} = \frac{1}{v_2} \frac{L - x}{\sqrt{(L-x)^2 + h_2^2}}$$

(4)

(c) Hence prove Snell’s Law which relates the optimum angles $\theta_1, \theta_2$ at the interface, namely:

$$\frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2}$$

(3)
Figure 3 shows a situation which is important in geophysics, where two seismic waves travel away from a source S. The direct wave travels just below the surface in the weathered layer, at a speed \( v_1 \). The refracted wave travels down to the bedrock, and then travels horizontally in the bedrock at a speed \( v_2 \). 

![Figure 3: Cross section showing direct (blue) and refracted (red) seismic waves. The weathered layer has thickness \( h \) and seismic velocity \( v_1 \). The bedrock has seismic velocity \( v_2 \). The angle \( \theta_1 \) which results in the refracted wave shown is called the critical angle.](image)

(d) According to Fermat’s Law, the refracted wave will travel the path of least time. Use Snell’s law to prove that the direction is governed by

\[
\sin \theta_1 = \frac{v_1}{v_2}
\]

(2)

(e) Write down an expression for the time taken for the direct wave to travel to the point \( P_1 \) at a horizontal distance \( x \) from \( S \)

(1)

(f) Write down an expression for the time taken for the refracted wave to travel to the point \( P_2 \), directly below \( P_1 \).

(3)

(g) If the weathering layer has a seismic velocity of \( v_1 = 1000 \text{ ms}^{-1} \) and the bedrock has a seismic velocity of \( v_2 = 3000 \text{ ms}^{-1} \), find the critical angle \( \theta_1 \).

(1)

(h) For the parameters given in (g), and for a weathering thickness of 20m, calculate the distance \( x \) at which the refracted wave overtakes the direct wave.

(4)
Appendix: Formulas and Constants

\[ G = 6.673 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \]
\[ M_E = 5.973 \times 10^{24} \text{ kg} \]
\[ R_E = 6371.0 \text{ km} \]
\[ \mu_0 = 1.257 \times 10^{-6} \text{ N A}^{-2} \]
\[ \delta g_L = 8.13 \sin 2\theta \text{ gu / km} \]
\[ \delta g_F = 3.086 \text{ gu / m} \]
\[ \delta g_B = 0.0004185\rho \text{ gu / m} \text{ (Note: } \rho \text{ in SI units)} \]