Q1. Seismic Wave Types and Elastic Constants

A number of different types of seismic waves can travel in and on the earth. The body waves are called P waves and S waves, whose transmission velocities are given by:

\[ v_P = \sqrt{\frac{k + \frac{4}{3} \mu}{\rho}} \]
\[ v_S = \sqrt{\frac{\mu}{\rho}} \]

(a) Suppose that a particular granite has incompressibility \( k \) of 40 GPa, rigidity \( \mu \) of 20 GPa, and density \( \rho \) of 2700 kg m\(^{-3}\). Show that the P-wave velocity is approximately 4970 m s\(^{-1}\). (Note: GPa = \( 10^9 \) Pascals, and is an SI unit.)

(b) For the same rock show that the S-wave velocity is about 2720 m s\(^{-1}\).

(c) Using your calculated values for \( v_P \) and \( v_S \) show that the P-wave to S-wave velocity ratio \( (\gamma) \) for this rock is about 1.826.

(d) By considering the above equations show that the velocity ratio \( (\gamma) \) is related to the basic elastic parameters \( k \) and \( \mu \) via the relationship:

\[ \gamma = \sqrt{\frac{k}{\mu} + \frac{4}{3}} \]

(e) Show that the observed value of \( \gamma \approx 1.826 \) can also be deduced by substituting the values for \( k \) and \( \mu \) in this expression.

(f) Another elastic parameter of particular interest to mining engineers and geologists is Poisson’s ratio \( (\sigma) \), which is related to the other elastic parameters via:

\[ \sigma = \frac{3k - 2\mu}{2(3k + \mu)} \]

Show that for this granite the Poisson’s Ratio is approximately 0.286.
(g) By manipulating the above expressions for \( v_P, v_S \) and \( \sigma \), Poisson’s ratio can also be related to the velocity ratio (\( \gamma \)) via:

\[
\sigma = \frac{\gamma^2 - 2}{2(\gamma^2 - 1)}
\]

By substituting the known value of \( \gamma \approx 1.826 \) show that the same value of Poisson’s ratio can be deduced.

(h) In another rock (a weathered shale) which is ‘weaker’ than granite, the P-wave and S-wave velocities as measured with the seismic refraction method are 2600 m s\(^{-1}\) and 1000 m s\(^{-1}\). Show that the velocity ratio is \( \gamma = 2.6 \) and the Poisson’s ratio is therefore approximately 0.41.

(i) You are a mining geologist responsible the monitoring the integrity of the roof of an underground mine. Based on the analysis of Parts (f)-(h), would you prefer your roof rocks to have:

- high or low \( \gamma \)
- high or low Poisson’s Ratio

Q2. Snell’s Law

(a) A P-wave hits the top of a coal seam at an incidence angle of 15°. The rock above the interface is a shale (\( v_P = 3000 \) m s\(^{-1}\), \( v_S = 1400 \) ms\(^{-1}\)). The coal has P-wave and S-wave velocities of 2000 ms\(^{-1}\) and 900 ms\(^{-1}\) respectively. Using Snell’s law show that:

- the reflected P-wave makes an angle of 15° with the normal
- the reflected S-wave makes an angle of 6.9° with the normal
- the transmitted P-wave makes an angle of 9.9° with the normal
- the transmitted S-wave makes an angle of 4.5° with the normal

(b) Use Snell’s law to show that a critical refraction cannot be generated at the top of the coal.
(c) Extending the model in Part (a), suppose that the coal is underlain by another shale body. Show that the directions taken by waves in the two shale bodies are related via:

\[
\frac{\sin i_1}{v_1} = \frac{\sin i_3}{v_3}
\]

where the indices 1 and 3 refer to the ‘roof’ and ‘floor’ shales respectively.

(d) Hence show that the condition for a critical refraction at the base of coal is given by:

\[
\sin i_1 = \frac{v_1}{v_3}
\]

(e) Suppose that the roof shale and coal have properties as in Part (a) and that the floor shale has P-wave and S-wave velocities of 3200 ms\(^{-1}\) and 1600 ms\(^{-1}\) respectively. Show that a P-wave which is incident on the top of coal at approximately 69.6° will generate a critical refraction at the base of coal.

(f) Show that the refraction off base coal makes an angle of approximately 38.7° while passing through the coal. Hence, sketch the path of this refracting P-wave, marking in angles in each layer.

(g) A P-wave has been generated from an airgun in a marine seismic survey. It hits the sea floor \((v_P = 2500 \text{ m s}^{-1}, v_S = 1200 \text{ ms}^{-1})\) at an incident angle of 20 °. Find the direction taken by each of the three waves leaving the interface (reflected and transmitted).

Q3. Reflection Coefficients

(a) Consider the shale-coal-shale model described in Q2. Suppose that the densities of the three formations are 2500 kg m\(^{-3}\), 1500 kg m\(^{-3}\), 2550 kg m\(^{-3}\) (from the top down). Show that the P-wave reflection coefficient for the top of coal is about -0.43.

(b) The reflection coefficient is the ratio of of the reflected amplitude to the incident amplitude, assuming approximately normal incidence. For the problem in Part (a), express the amplitude of the reflected wave as a percentage of the incident wave, and comment on the polarity (i.e. normal or reversed) of the reflected wave.

(c) Show that the reflection coefficient from the base-of-coal is slightly larger in magnitude, but does not produce a polarity reversal.
Q4. Seismic Refraction

(a) A common problem in hydrocarbon exploration, engineering and environmental geology is to determine the properties of the weathering layer. Show that if the weathering layer is approximately horizontal, with a thickness $Z_1$ and a seismic velocity $v_1$ then the travel time ($t$) vs distance ($x$) relationships for the direct wave (P-wave along surface) and the critically refracted wave are given by:

$$t_D = \frac{x}{v_1}$$

$$t_R = \frac{x}{v_2} + \frac{2Z_1 \cos i_c}{v_1}$$

where $v_2$ is the velocity in the sub-weathering layer, and $i_c$ is the critical angle.

(b) With reference to the equation $t = mx + C$ explain how the velocities of the weathering and sub-weathering can be determined from slopes on a plot of travel time vs distance.

(c) With reference to the equation $t = mx + C$ explain how the thickness of the weathering layer can be determined from an intercept on a plot of travel time vs distance.

(d) On a field plot of travel time ($t$) versus distance ($x$), the velocities of the layers are determined (from measurements of 1/slope) as 500 ms$^{-1}$ and 2500 ms$^{-1}$ respectively. The intercept of the refraction segment is measured as 65 ms. Show that the critical angle is about 11.5$^\circ$, and hence show that the thickness of weathering is approximately 16.6m.

(e) Since the velocity contrast is strong, then a reasonable approximation to the weathering depth is obtained by approximating the factor $\cos i_c$ by 1.0. Show that this leads to an error of about -2% in the weathering depth.

Q5. Seismic Reflection: NMO Equation

(a) Sketch a seismic ray reflecting off an interface at a depth $Z$, and travelling to a geophone at a distance $x$ from the source.

(b) Using Pythagoras’ Theorem, show that the total path length of this reflected ray ($d$) is

$$d = 2\sqrt{\frac{x^2}{4} + Z^2}$$

(c) If the seismic velocity in the region above the reflector is $v$, show that the travel time
(t_x) for the reflected wave to the geophone at distance x obeys

\[ t_x^2 = \frac{4Z^2}{v^2} + \frac{x^2}{v^2} \]

(d)
The reflection time for a geophone right next to the source is called the zero-offset time \( t_0 \). Show that the zero-offset reflection time is \( t_0 = \frac{2Z}{v} \)

(e)
By combining the results in (c) and (d) deduce the so called Normal Moveout (NMO) equation which relates the reflection time for a geophone at offset \( x \) to that for a geophone at zero offset.

\[ t_x^2 = t_0^2 + \frac{x^2}{v^2} \]

(f)
Show that the NMO equation can be expressed as

\[ \frac{t_x^2}{A^2} - \frac{x^2}{B^2} = 1 \]

where \( A = t_0 \) and \( B = t_0v \)

(g)
Google *equation of hyperbola* and hence convince yourself that seismic reflection times obey a hyperbolic relationship with offset (distance).

**Q6. Seismic Reflection: Practical NMO Analysis**

(a)
A seismic shot record is a plot of \( t_x \) vs \( x \). Based on the analysis in Part 4(f) a reflection event should appear hyperbolic on a shot record. On the accompanying shot record, mark in the first strong reflection event (around 0.8s at the centre of the record). Be careful to keep to the same event, and try to mark right to the edge of the record.

(b)
The vertical axis is time in seconds. Carefully estimate the zero-offset time \( t_0 \) for your marked reflection event (i.e. the time at the centre of the record, close to the source location).

(c)
Estimate the reflection time \( t_x \) for a geophone at the maximum offset (distance) from the source (i.e. at the extreme edge of the record).

(d)
The geophone spacing on this record is 25m. Show that the offset \( x \) in metres of the geophone at the maximum offset is 1500m.
(e) Show that the NMO equation can be re-arranged to allow estimation of the average velocity above the reflector, namely:

\[ v = \sqrt{\frac{x^2}{t_x^2 - t_0^2}} \]

(f) Substitute your measured values of \( t_0, t_x, \) and \( x, \) and hence estimate the average seismic velocity \( (v) \) in the earth above your reflector. (Depending on exactly where you have marked, you should get a velocity in the region of 2700-2800 ms\(^{-1}\)).

(g) Mark in a deeper reflector (e.g. the one with \( t_0 \approx 1.8 \) s.) Repeat the exercise to estimate the average velocity down to this reflector. You should get a higher velocity (around 3200 ms\(^{-1}\)).

(h) Why would you expect average velocities to increase with depth?